Bargaining Over the Interference Channel

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Abstract

In this paper we analyze the interference channel as a conflict situation. This viewpoint implies that certain points in the rate region are unreasonable to one of the players. Therefore these points cannot be considered achievable based on game theoretic considerations. We then propose to use Nash bargaining solution as a tool that provides preferred points on the boundary of the game theoretic rate region. We provide analysis for the 2x2 intereference channel using the FDM achievable rate region. We also outline how to generalize our results to other achievable rate regions for the interference channel as well as the multiple access channel.

Keywords: Spectrum optimization, distributed coordination, game theory, interference channel, multiple access channel.

I. Introduction

Computing the capacity region of the interference channel is an open problem in information theory [1]. A good overview of the results until 1985 is given by van der Meulen [2] and the references therein. The capacity region of general interference case is not known yet. However, in the last forty five years of research some progress has been made. Ahslswede [3], derived a general formula for the capacity region of a discrete memoryless Interference Channel (IC) using a limiting expression which is computationally infeasible. Cheng, and Verdu [4] proved that the limiting expression cannot be written in general by a single-letter formula and the restriction to Gaussian inputs provides only an inner bound to the capacity region of the IC. The best known achievable region for the general interference channel is due to Han and Kobayashi [5]. However the computation of the Han and Kobayashi formula for a general discrete memoryless channel is in general too complex. Sason [7] describes certain improvement over the Han Kobayashi rate region in certain cases. In this paper we focus on the 2x2 memoryless Gaussian interference channel. A 2x2 Gaussian interference channel in standard form (after suitable normalization) is given by:

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where

$$\mathbf{H} = \begin{bmatrix} 1 & \alpha \\ \beta & 1 \end{bmatrix}$$

$\mathbf{s} = [s_1, s_2]^T$, and $\mathbf{x} = [x_1, x_2]^T$ are sampled values of the input and output signals, respectively. The noise vector $\mathbf{n}$ represents the additive Gaussian noises with zero mean and unit variance. The powers of the input signals are constrained to be less than P_1, P_2 respectively. The off-diagonal elements of $\mathbf{H}$, α, β represent the degree of interference present. The capacity region of the Gaussain interference channel with very strong interference (i.e., $\alpha \geq 1 + P_1$, $\beta \geq 1 + P_2$) was found by Carleial given by

$$R_i \leq \log_2(1 + P_i), \quad i = 1, 2. \tag{2}$$

This surprising result shows that very strong interference dose not reduces the capacity. A Gaussian interference channel is said to have strong interference if $\min\{\alpha, \beta\} > 1$. Sato [6] derived an achievable capacity region (inner bound) of Gaussian interference channel as intersection of two multiple access gaussian capacity regions embedded in the interference channel. The achievable region is the subset of the rate pair of the rectangular region of the very strong interference (1 2) and the region

$$R_1 + R_2 \leq \log_2 \left(\min\left\{1 + P_1 + \alpha P_2, 1 + P_2 + \beta P_1\right\}\right) \tag{3}$$

A recent progress for the case of Gaussian interference is described by Sason [7]. Sason derived an achievable rate region based on a modified time- (or frequency-) division multiplexing approach which was originated by Sato for

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the degraded Gaussian IC. The achievable rate region includes the rate region which is achieved by time/frequency division multiplexing (TDM/ FDM), and it also includes the rate region which is obtained by time sharing between the two rate pairs where one of the transmitters sends its data reliably at the maximal possible rate (i.e., the maximum rate it can achieve in the absence of interference), and the other transmitter decreases its data rate to the point where both receivers can reliably decode their messages.

In this paper we limit ourselves to the frequency-division multiplexing (FDM) scheme where an assignment of disjoint portions of the frequency band to the several transmitters is made. This technique is widely used in practice because simple filtering can be used at the receivers to eliminate interference. The results equivalently apply to time-division multiplexing (TDM). In both cases we use the non-naive version, where all power is used in the frequency/time slice allocated for a given user.

While information theoretical considerations allow all points in the rate region, we argue that the interference channel is a conflict situation between the interfering links. Each link is considered a player in a general interference game. As such it has been shown that non-cooperative solutions such as the iterative water-filling, which leads to good solutions for the multiple access channel (MAC) and the broadcast channel [8] can be highly suboptimal in interference channel scenarios [9], [10]. To solve this problem there are several possible approaches. One that has gained popularity in recent years is through the use of competitive strategies in repeated games [11], [12]. Our approach is significantly different and is based on general bargaining theory originally developed by Nash. We claim that while all points on the boundary of the interference channel are achievable from the strict informational point of view, most of them will never be achieved since one of the players will refuse to use coding strategies leading to these points. The rates of interest are only rates that are higher than the rates that each user can achieve, independently of the other user coding strategy. Such a rate pair must form a Nash equilibrium [13]. This implies that not all the rates achievable from pure information theoretic point of view are indeed achievable from game theoretic prespective. Hence we define the game theoretic rate region.

Definition 1.1: Let $\mathcal{R}$ be an achievable information theoretic rate region. The game theoretic rate region $\mathcal{R}^G$ is given by

$$\mathcal{R}^G = \{(R_1, R_2) \in \mathcal{R} : R_i^c \leq R_i, \quad i = 1, 2\} \tag{4}$$

where R_i^c is the rate achievable by user i in a non-cooperative interference game.
To see what are the pair rates that can be achieved by negotiation of the two users we resort to a well known solution termed the Nash bargaining solution. In his seminal papers [14], [15], Nash proposed 4 axioms that a solution to a bargaining problem should satisfy. He then proves that there exists a unique solution satisfying these axioms. We will analyze the application of Nash bargaining solution (NBS) to the interference game, and show that there exists a unique point on the boundary of the capacity region which is the solution to the bargaining problem as posed by Nash.

The fact that the Nash solution can be computed independently by users, using only channel state information, provides a good method for managing multi-user ad-hoc networks operating in an unregulated environment.

Due to space limitations the paper considers only the Gaussian interference channel and FDM strategy suitable for medium interference [16]. However extensions to other achievable rate regions of the interference channels will appear in a subsequent paper.

Application of Nash bargaining to OFDMA has been proposed by [17]. However in that paper the solution was used only as a measure of fairness. Therefore R_i^c was not taken as the Nash equilibrium for the competitive game, but an arbitrary $R_i^{\min}$. This can result in non-feasible problem, and the proposed algorithm might be unstable. Furthermore our approach can be extended to other coding strategies such as in [7].

II. Nash equilibrium vs. Nash Bargaining solution

In this section we describe to solution concepts for 2 players games. The first notion is that of Nash equilibrium. The second is the Nash bargaining solution (NBS). In order to simplify the notation we specifically concentrate on the Gaussian interference game.

A. The Gaussian interference game

In this section we define the Gaussian interference game, and provide some simplifications for dealing with discrete frequencies. For a general background on non-cooperative games we refer the reader to [18] and [19].

The Gaussian interference game was defined in [20]. In this paper we use the discrete approximation game. Let $f_0 < \cdots < f_K$ be an increasing sequence of frequencies. Let I_k be the closed interval be given by $I_k = [f_{k-1}, f_k]$. We now define the approximate Gaussian interference game denoted by $GI_{\{I_1,\ldots,I_K\}}$.

Let the players $1,\ldots,N$ operate over separate channels. Assume that the N channels have crosstalk coupling functions $h_{ij}(k)$. Assume that user i'th is allowed to transmit a total power of P_i. Each player can transmit a power vector $\mathbf{p}_i = (p_i(1),\ldots,p_i(K)) \in [0,P_i]^K$ such that $p_i(k)$ is the power transmitted in the interval I_k. Therefore we have $\sum_{k=1}^K p_i(k) = P_i$. The equality follows from the fact that in non-cooperative scenario all users will use the maximal power they can use. This implies that the set of power distributions for all users is a closed convex subset of the cube $\prod_{i=1}^N [0,P_i]^K$ given by:

$$\mathbf{B} = \prod_{i=1}^N \mathbf{B}_i \tag{5}$$

where $\mathbf{B}_i$ is the set of admissible power distributions for player i is

$$\mathbf{B}_i = [0,P_i]^K \cap \left\{ (p(1),\ldots,p(K)) : \sum_{k=1}^K p(k) = P_i \right\} \tag{6}$$

Each player chooses a PSD $\mathbf{p}_i = \langle p_i(k) : 1 \le k \le N \rangle \in \mathbf{B}_i$. Let the payoff for user i be given by:

$$C^i(\mathbf{p}_1,\ldots,\mathbf{p}_N) = \sum_{k=1}^K \log_2\left(1 + \frac{|h_i(k)|^2 p_i(k)}{\sum |h_{ij}(k)|^2 p_j(k) + \mathbf{n}(k)}\right) \tag{7}$$

where C^i is the capacity available to player i given power distributions $\mathbf{p}_1,\ldots,\mathbf{p}_N$, channel responses $h_i(f)$, crosstalk coupling functions $h_{ij}(k)$ and $n_i(k) > 0$ is external noise present at the i'th channel receiver at frequency k. In cases where $n_i(k) = 0$ capacities might become infinite using FDM strategies, however this is non-physical situation due to the receiver noise that is always present, even if small. Each C^i is continuous on all variables.

Definition 2.1: The Gaussian Interference game $GI_{\{I_1,\ldots,I_k\}} = \{\mathbf{C},\mathbf{B}\}$ is the N players non-cooperative game with payoff vector $\mathbf{C} = (C^1,\ldots,C^N)$ where C^i are defined in (7) and $\mathbf{B}$ is the strategy set defined by (5).
The interference game is a special case of non-cooperative N-persons game.

B. Nash equilibrium in non-cooperative games

An important notion in game theory is that of a Nash equilibrium.

Definition 2.2: An N-tuple of strategies $\langle \mathbf{p}_1,\ldots,\mathbf{p}_N \rangle$ for players $1,\ldots,N$ respectively is called a Nash equilibrium iff for all n and for all $\mathbf{p}$ ($\mathbf{p}$ a strategy for player n)

$$C^n(\mathbf{p}_1,\ldots,\mathbf{p}_{n-1},\mathbf{p},\mathbf{p}_{n+1},\ldots,\mathbf{p}_N) < C^n(\mathbf{p}_1,\ldots,\mathbf{p}_N)$$

i.e., given that all other players $i \neq n$ use strategies $\mathbf{p}_i$, player n best response is $\mathbf{p}_n$.
The proof of existence of Nash equilibrium in the general interference game follows from an easy adaptation of the proof of the this result for convex games [21]. A much harder problem is the uniqueness of Nash equilibrium points in the water-filling game. This is very important to the stability of the waterfilling strategies. A first result in this direction has been given in [22]. A more general analysis of the convergence (although it still does not cover the case of arbitrary channels) has been given in [23].

C. Nash bargaining solution for the interference game

Nash equilibria are inevitable whenever non-cooperative zero sum game is played. However they can lead to substantial loss to all players, compared to a cooperative strategy in the non-zero sum case, where players can cooperate. Such a situation is called the prisoner's dilemma. The main issue in this case is how to achieve the cooperation in a stable manner and what rates can be achieved through cooperation.

In this section we present the Nash bargaining solution [14], [15]. The underlying structure for a Nash bargaining in an N players game is a set of outcomes of the bargaining process S which is compact and convex. S can be considered as a set of possible joint strategies or states, a designated disagreement outcome d (which represents the agreement to disagree and solve the problem competitively) and a multiuser utility function $U : S \cup \{d\} \rightarrow \boldsymbol{R}^N$.

The Nash bargaining is a function F which assigns to each pair $(S \cup \{d\}, U)$ as above an element of $S \cup \{d\}$. Furthermore, the Nash solution is unique. In order to obtain the solution, Nash assumed four axioms:

Linearity. This means that if we perform the same linear transformation on the utilities of all players than the solution is transformed accordingly.

Independence of irrelevant alternatives. This axiom states that if the bargaining solution of a large game $T \cup \{d\}$ is obtained in a small set S. Then the bargaining solution assigns the same solution to the smaller game, i.e., The irrelevant alternatives in $T \backslash S$ do not affect the outcome of the bargaining.

Symmetry. If two players are identical than renaming them will not change the outcome and both will get the same utility.

Pareto optimality. If s is the outcome of the bargaining then no other state t exists such that $U(s) < U(t)$ (coordinate wise).

A good discussion of these axioms can be found in [18]. Nash proved that there exists a unique solution to the bargaining problem satisfying these 4 axioms. The solution is obtained by maximizing

$$s = \arg \max_{s \in S \cup \{d\}} \prod_{n=1}^{N} \left(U_i(s) - U_i(d) \right) \tag{8}$$

Typically one assumes that there exist at least one feasible $s \in S$ such that $U(d) < U(s)$ coordinatewise, but otherwise we can assume that the bargaining solution is d. In our case the utility for user i is given by the rate R_i, and $U_i(d)$ is the competitive Nash equilibrium, obtained by iterative waterfilling for general ISI channels.

III. Existence and uniqueness of Nash bargaining solution for the two players interference game

In this section we outline the proof that a Nash bargaining solution always exists for utility function given by capacity for any achievable rate region for the 2x2 interference channel.

An achievable rate region can always be defined by the following equations:

$$\begin{array}{c} 0 \le R_1 \le R^1_{\max} \\ 0 \le R_2 \le f(R_1) \end{array} \tag{9}$$

where $f(R)$ is a monotonically decreasing concave function of R_1. The monotonicity is obvious and the concavity follows from a standard time sharing argument.

Theorem 3.1: Assume that we are given an achievable rate region for the interference channel described by (9). Let $U_i(R) = R$ be the utility of the i'th user, and let R_i^c be the achievable rate at the competitive Nash equilibrium point for the i'th channel. Then there is a unique point $\left(R_1^{NBS}, R_2^{NBS}\right)$ that is the Nash bargaining solution using the encoding strategies of the given achievable rate region for the interference channel. This point is Pareto optimal and therefore on the boundary of the rate region, i.e., $R_2 = f(R_1)$.

Note that by concavity f is strictly decreasing, except on an initial segment of rates for player I. Due to space limitations, full proof of this theorem will be provided in an extended version of this paper. However the following example provides the intuition underlying theorem 3.1, the relation between the competitive solution, the NBS and the game theoretic rate region $\mathcal{R}^G$ we have chosen $\mathrm{SNR}_1 = 20$ dB, $\mathrm{SNR}_2 = 15$ dB, and $\alpha = 0.4, \beta = 0.7$. Figure 1 presents the FDM rate region, the Nash equilibrium point denoted by , and a contour plot of $F(\rho)$. It can be seen that the convexity of $F(\rho)$ together with the concavity of the function defining the upper oundary of the rate region implies that at there is a unique contour tangent to the rate region. The tangent point is the Nash bargaining solution. We can see that the NBS achieves rates that are 1.6 and 4 times higher than the rates of the competitive Nash equilibrium rates for player I and player II respectively. The game theoretic rate region is the intersection of the information theoretic rate region with the quadrant above the dotted lines. Finally we comment that this theorem can be generalized to N players with higher notational complexity.

IV. Bargaining for the two players interference game

In this section we analyze the two players interference game, with TDM/FDM strategies. We provide conditions under which the bargaining solution provides improvement over the competitive solution. This extends the work of [9] where it is characterized when does FDM solution outperforms the competitive IWF solution for symmetric

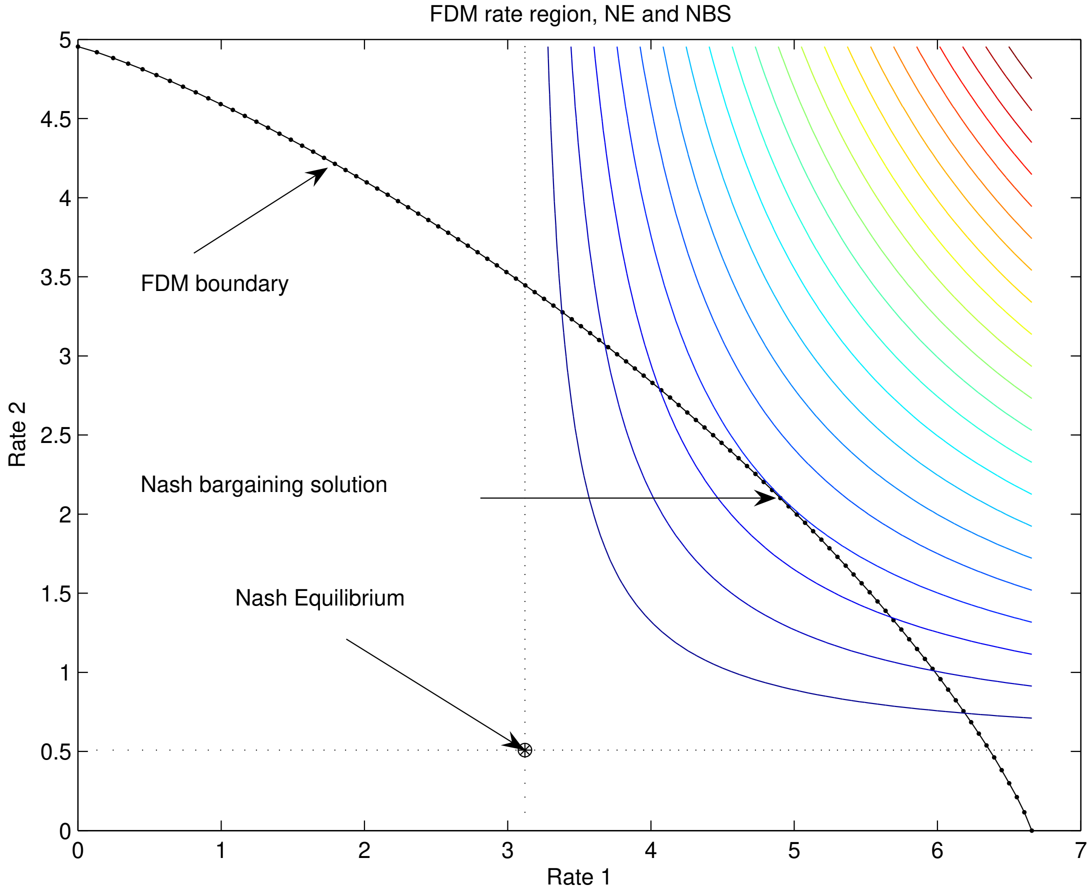

Fig. 1. FDM rate region (thick line), Nash equilibrium $*$, Nash bargaining solution and the contours of $F(\rho)$. $\mathrm{SNR}_1 = 20$ dB, $\mathrm{SNR}_2 = 15$ dB, and $\alpha = 0.4, \beta = 0.7$

2x2 interference game. We assume that the utility of player i is given by $U_i = R_i$, the achievable rate. To that end lets consider the general 2x2 interference channel (in non-standard form). The received signal vector $\mathbf{x}$ is given by

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{10}$$

where $\mathbf{x} = [x_1, x_2]^T$ is the received signal,

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{11}$$

is the channel matrix and $\mathbf{s} = [s_1, s_2]^T$ is the vector of transmited signals. Note that in our cases both transmission and reception are performed independently, and the vector formulation is for notational simplicity only. Similarly to the analysis of [9] the competitive strategies in the interference game are given by flat power allocation resulting in rates:

$$\begin{array}{c} R_1^c = \frac{W}{2}\log_2\left(1 + \frac{|h_{11}|^2 P_1}{WN_0/2 + |h_{12}|^2 P_2}\right) \\ R_2^c = \frac{W}{2}\log_2\left(1 + \frac{|h_{22}|^2 P_2}{WN_0/2 + |h_{21}|^2 P_1}\right) \end{array} \tag{12}$$

Dividing by the noise power $WN_0/2$ we obtain

$$\begin{array}{c} R_1^c = \frac{W}{2}\log_2\left(1 + \frac{\mathrm{SNR}_1}{1+\alpha\mathrm{SNR}_2}\right) \\ R_2^c = \frac{W}{2}\log_2\left(1 + \frac{\mathrm{SNR}_2}{1+\beta\mathrm{SNR}_1}\right) \end{array} \tag{13}$$

where

$$\mathrm{SNR}_i = \frac{|h_{ii}|^2 P_i}{WN_0/2}, \qquad \alpha = \frac{|h_{12}|^2}{|h_{22}|^2}, \qquad \beta = \frac{|h_{21}|^2}{|h_{11}|^2}$$

This is equivalent ot the standard channel (1), with $P_i = \text{SNR}_i$. Since the rates R_i^c are achieved by competitive strategy, player i would not cooperate unless he will obtain a rate higher than R_i^c. Therefore the game theoretic rate region is defined by pair rates higher that R_i^c of equation (13).

Since we are interested in FDM cooperative strategies assume that player I uses a fraction of $0 \le \rho \le 1$ of the band and user II uses a fraction $1-\rho$. The rates obtained by the two users are given by

$$\begin{array}{l} R_1(\rho) = \frac{\rho W}{2}\log_2\left(1+\frac{\text{SNR}_1}{\rho}\right) \\ R_2(1-\rho) = \frac{(1-\rho)W}{2}\log_2\left(1+\frac{\text{SNR}_2}{1-\rho}\right) \end{array} \tag{14}$$

The two users will benefit from FDM type of cooperation as long as

$$\begin{array}{l} R_i^c \le R_i(\rho_i), \quad i=1,2 \\ \rho_1+\rho_2 \le 1 \end{array} \tag{15}$$

For each $0 < x, y$ define $f(x,y)$ as the minimal ρ that solves the equation

$$\left(1+\frac{x}{\rho}\right)^{\rho} = 1+\frac{x}{1+y} \tag{16}$$

Claim 4.1: $f(x,y)$ is a well defined function for $x, y \in \boldsymbol{R}^+$.
Proof: Let

$$g(x,y,\rho) = \left(1+\frac{x}{\rho}\right)^{\rho} - 1 - \frac{x}{1+y}$$

For every x, y, $g(x,y,\rho)$ is a continuous and monotonic function in ρ. Furthermore, for any $0 \le x, y$, $g(x,y,1) > 0$, while

$$\lim_{\rho\to 0} g(x,y,\rho) < 0.$$

so there is a unique solution to (16).

Claim 4.2: Assume now that

$$f(\text{SNR}_1, \alpha\text{SNR}_2) + f(\text{SNR}_2, \beta\text{SNR}_1) \le 1. \tag{17}$$

Then an FDM Nash bargaining solution exists. The NBS is given by solving the problem

$$\max_{\rho} F(\rho) = \max_{\rho} \tag{18}$$

where

$$F(\rho) = \left(R_1(\rho) - R_1^c\right)\left(R_2(1-\rho) - R_2^c\right) \tag{19}$$

and $R_i(\rho)$ are defined by (14).
Proof: Let $\rho_1 = f(\text{SNR}_1, \alpha\text{SNR}_2), \rho_2 = f(\text{SNR}_2, \beta\text{SNR}_1)$. By definition of f player i has the same rate as the competitive rate if he can use a ρ_i, fraction of the bandwidth. Since (17) implies that $\rho_1+\rho_2 \le 1$ FDM is preferable to the competitive solution.

A special case can now be derived:

Claim 4.3: Assume that $\text{SNR}_1 \ge \frac{1}{2}\left(\alpha^2\beta^4\right)^{-1/3}$ and $\text{SNR}_2 \ge \frac{1}{2}\left(\beta^2\alpha^4\right)^{-1/3}$. Then there is a Nash bargaining solution that is better than the competitive solution.
The proof of the claim follows directly by substituting $\rho_1 = \rho_2 = 1/2$.

We also provide without proof the asymptotic performance as SNR_i increases to infinity

Claim 4.4: For each i and for any fixed z, $\lim_{SNR_i \to \infty} f(\text{SNR}_i, z) = 0$.
This implies that if one of the users has sufficiently high SNR than FDM strategy is preferable to competitive strategy. This fact will be evident from the simulations in the next section.

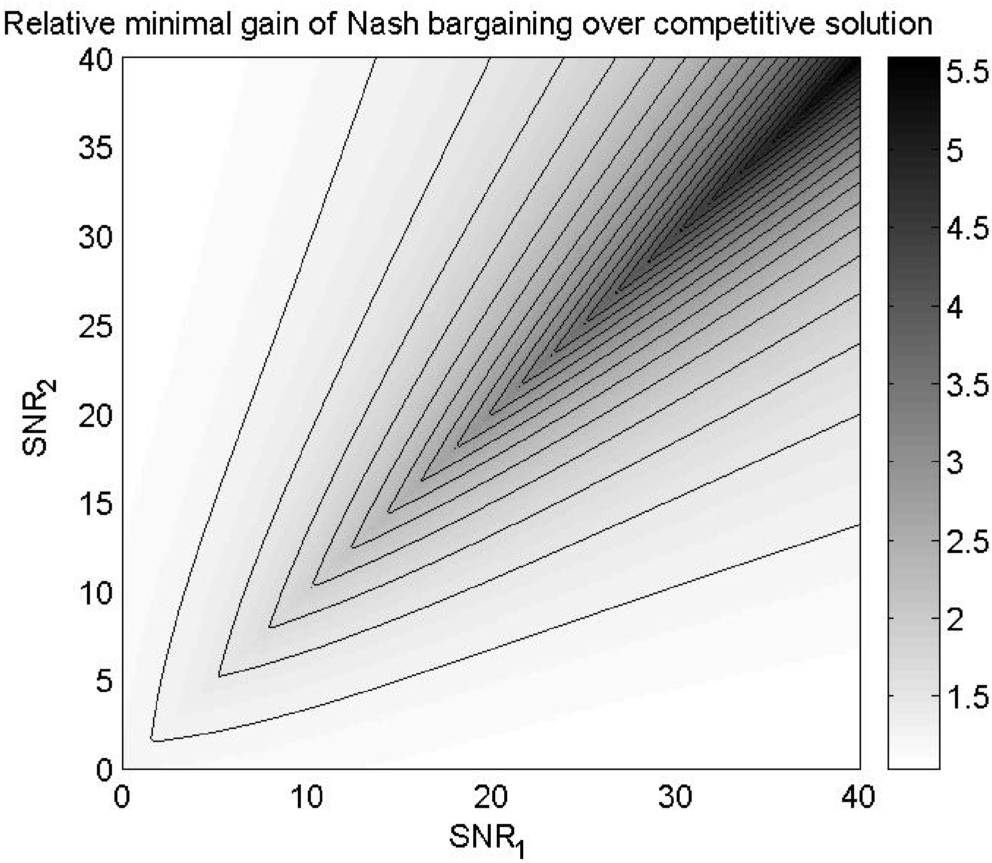

Fig. 2. Minimal relative improvement. $\alpha = \beta = 0.7$.

V. Simulations

In this section we compare in simulations the Bargaining solution to the competitive solution for various situations with medium interference. We have tested the gain of the Nash bargaining solution relative to the Nash equilibrium competitive rate pair as a function of channel coefficients as well as signal to noise ratio. To that end we define the minimum relative improvement by:

$$\Delta_{\min} = \min \left\{ \frac{R_1^{NBS}}{R_1^c}, \frac{R_2^{NBS}}{R_2^c} \right\} \tag{20}$$

and the sum rate improvement by

$$\Delta_{sum} = \frac{R_1^{NBS} + R_2^{NBS}}{R_1^c + R_2^c} \tag{21}$$

In the first set of experiments we have fixed α, β and varied $\mathrm{SNR}_1, \mathrm{SNR}_2$ from 0 to 40 dB in steps of 0.25dB. Figure 2 presents $\Delta_{\min}$ for an interference channel with $\alpha = \beta = 0.7$. We can see that for high SNR we obtain significant improvement. Figure 3 presents the relative sum rate improvement Δ_{sum} for the same channel. We can see that the achieved rates are 5.5 times those of the competitive solution. We have now studied the effect of the interference coefficients on the Nash Bargaining solution. We have set the signal to additive white Gaussian noise ratio for both users to 20 dB, and varied α and β between 0 and 1. Similarly to the previous case we present the minimal improvement $\Delta_{\min}$ and the sum rate improvement Δ_{sum}. The results are shown in figures 4,5.

VI. Conclusions

In this paper we have defined the game theoretic rate region for the interference channel. The region is a subset of the rate region of the interference channel. We have shown that a specific point in the rate region given by the Nash

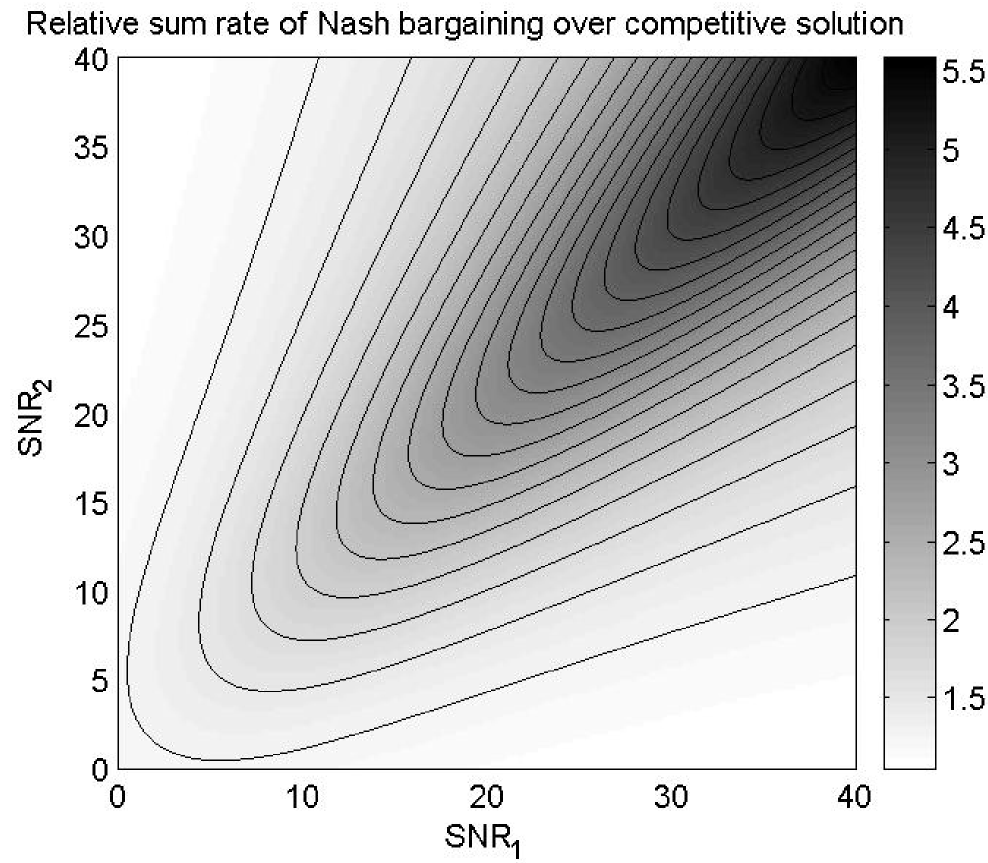

Fig. 3. sum rate relative improvement. $\alpha = \beta = 0.7$.

bargaining solution is better than other points in the context of bargaining theory. We have shown conditions for the existence of such a point in the case of the FDM rate region. Finally we have demonstrated through simulations the significant improvement of the cooperative solution over the competitive Nash equilibrium.

References

[1] T.M. Cover and J. A. Thomas, *Elements of Information Theory*. New York, NY: John Wiley and Sons, 1991.

[2] E.C. van der Meulen, "Some reflections on the interference channel," in *Communications and Cryptography: Two Sides of One Tapestry* (R.E. Blahut, D. J. Costell, and T. Mittelholzer, eds.), pp. 409–421, Kluwer, 1994.

[3] R. Ahlswede, "Multi-way communication channels," in *Proceedings of 2nd International Symposium on Information Theory*, pp. 23–52, Sept. 1973.

[4] R.S. Cheng and S. Verdu, "On limiting characterizations of memoryless multiuser capacity regions," *IEEE Trans. on Information Theory*, vol. 39, pp. 609–612, Mar. 1993.

[5] T.S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. on Information Theory*, vol. 27, pp. 49–60, Jan. 1981.

[6] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. on Information Theory*, vol. 27, pp. 786–788, nov 1981.

[7] I. Sason, "On achievable rate regions for the Gaussian interference channel," *IEEE Trans. on Information Theory*, vol. 50, pp. 1345–1356, June 2004.

[8] W. Yu, W. Rhee, S. Boyd, and J.M. Cioffi, "Iterative waterfilling for Gaussian vector multiple-access channels," *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 145–152, 2004.

[9] A. Laufer and A. Leshem, "Distributed coordination of spectrum and the prisoner's dilemma," in *Proc. of the First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks - DySPAN 2005*, pp. 94 – 100, 2005.

[10] A. Laufer, A. Leshem, and H. Messer, "Game theoretic aspects of distributed spectral coordination with application to DSL networks," *IEEE Trans. on Informatiom Theory*, 2005. Submitted.

[11] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," in *Proc. of the First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks - DySPAN 2005*, pp. 251 – 258, 2005.

[12] N. Clemens and C. Rose, "Intelligent power allocation strategies in an unlicensed spectrum," in *Proc. of the First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks - DySPAN 2005*, pp. 37–42, 2005.

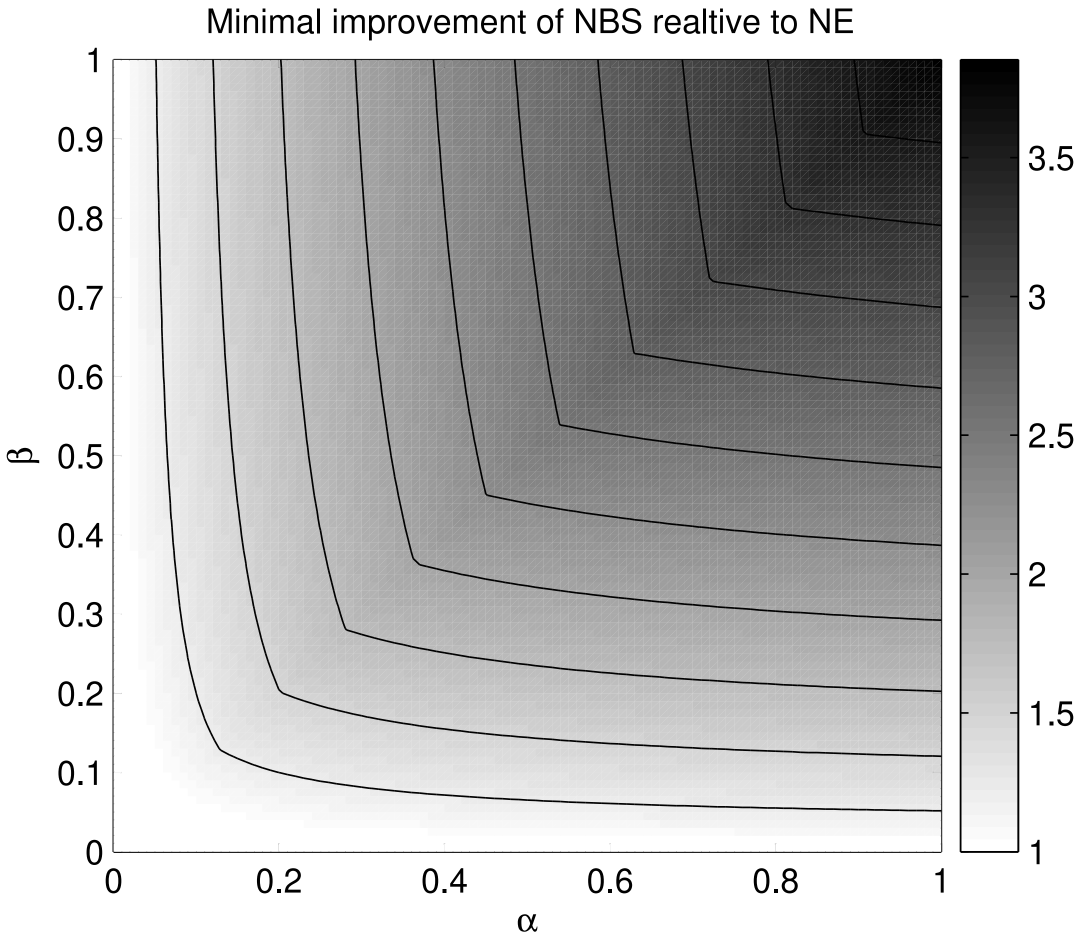

Fig. 4. Minimal relative improvement. SNR=20 dB.

[13] J. Nash, "Non-cooperative games," *The Annals of Mathematics*, vol. 54, pp. 286–295, Sept. 1951.

[14] J. Nash, "The bargaining problem," *Econometrica*, vol. 18, pp. 155–162, Apr. 1950.

[15] J. Nash, "Two-person cooperative games," *Econometrica*, vol. 21, pp. 128–140, Jan. 1953.

[16] M.H.M. Costa, "On the Gaussian interference channel," *IEEE Trans. on Information Theory*, vol. 31, pp. 607–615, Sept. 1985.

[17] Z. Han, Z. Ji, and K.J.R. Liu, "Fair multiuser channel allocation for OFDMA networks using the Nash bargaining solutions and coalitions," *IEEE Trans. on Communications*, vol. 53, pp. 1366–1376, Aug. 2005.

[18] G. Owen, *Game theory*. Academic Press, third ed., 1995.

[19] T. Basar and G.J. Olsder, *Dynamic non-cooperative game theory*. Academic Press, 1982.

[20] W. Yu, G. Ginis, and J.M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE Journal on Selected areas in Communications*, vol. 20, pp. 1105–1115, june 2002.

[21] H. Nikaido and K. Isoida, "Note on non-cooperative convex games," *Pacific Journal of Matematics*, vol. 5, pp. 807–815, 1955.

[22] S.T. Chung, J. Lee, S.J. Kim, and J.M. Cioffi, "On the convergence of iterative waterfilling in the frequency selective Gaussian interference channel," *Preprint*, 2002.

[23] Z.-Q. Luo and J.-S. Pang, "Analysis of iterative waterfilling algorithm for multiuser power control in digital subscriber lines," *EURASIP Journal on Applied Signal Processing on Advanced Signal Processing Techniques for Digital Subscriber Lines*.

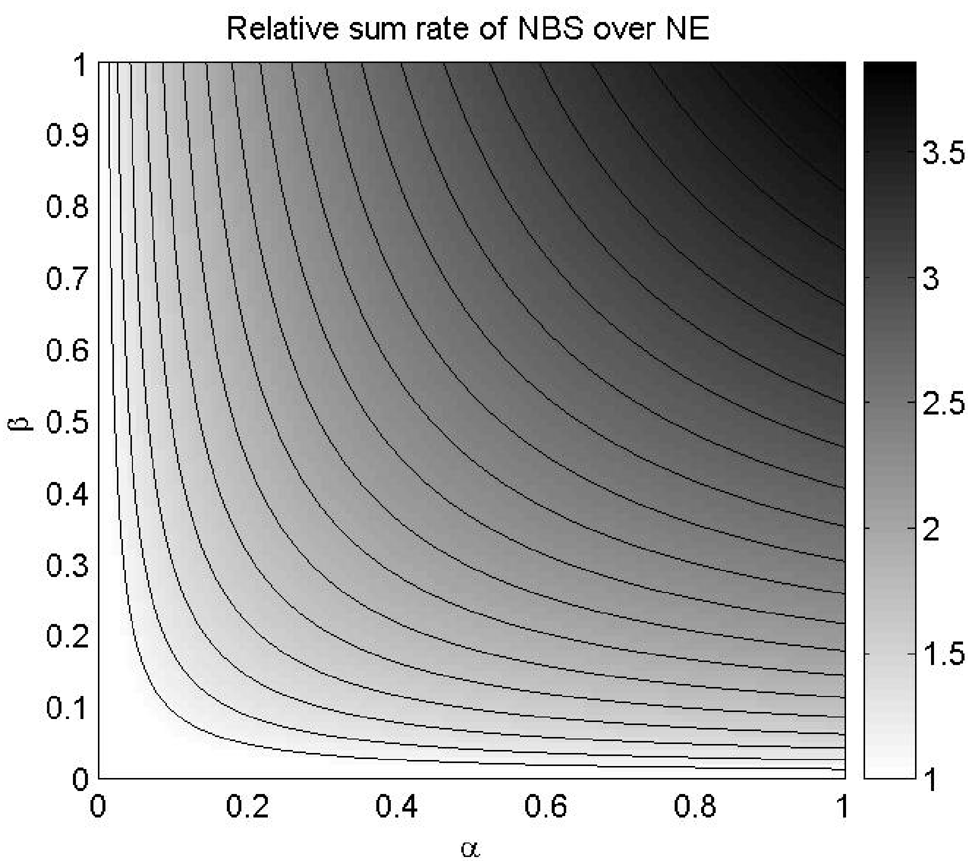

Fig. 5. Sum rate relative improvement. SNR=20 dB.