A Cost-Effective Maximum Likelihood Receiver for Multicarrièr Systems

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Abstract

Equalization structures for maximum likelihood (ML) reception of data transmitted over intersymbol interference channels are studied in this paper. Complexity constraints that are appropriate for multicarrier demodulators are imposed in the derivation of the corresponding front-end equalization structures that maximize a measure of performance for ML receivers. The equalizer that is best for the ML receiver is derived from a general theory of decision-aided equalization.

While a theory of decision-aided equalization is used, the resulting optimum equalizers are linear and do not use previous decisions. If the equalizer complexity is permitted to be infinite (but the ML detector complexity is finite), then a general optimum class of structures is derived that includes the well-known decision feedback equalizer (DFE) and the lesser-known autoregressive moving average (ARMA) filters. When a complexity constraint is also imposed on the equalizer, one of the structures in this class will be best for a given ML receiver. The best structure is found by a simple search procedure given herein. Our results indicate that near-optimum performance can be achieved using the approach in this paper at a great computational reduction.

1 Introduction

Maximum likelihood (ML) receivers that minimize probability of error in data transmission over channels with intersymbol interference can be realized in two popular forms, multicarrier demodulators or sequence (Viterbi) detectors. Multicarrier modulation/demodulation methods have long been known as optimum on the intersymbol interference channel [1] because they directly implement maximum likelihood (ML) detection for the given transmitted signal (which is a sum of independently modulated subcarrier signals, whence the name "multicarrier"). Multicarrier methods require an infinite number of subcarriers to be used to achieve data rates near capacity [2]. Sequence (Viterbi) detectors are comparatively less ancient [3], and can be used to implement maximum-likelihood detection for single-carrier or baseband data transmission systems. Sequence detectors also require an infinite number of states to decode transmitted signals at rates near capacity on a bandlimited (or intersymbol interference) channel. The transmission engineer is then invariably

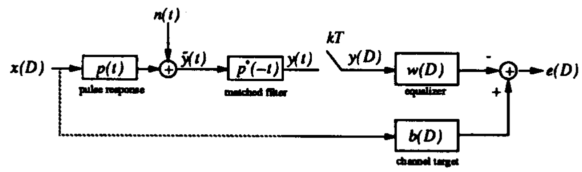

Figure 1: Input-assisted equalization.

left with the problem of trying to design a transmission system that exhibits performance as close to optimum as possible for given finite cost of implementation. The solution to this problem often leads to approximation of ML performance with a finite-complexity receiver. This paper focuses on one approach to this problem that approximates optimum performance with a significantly reduced complexity receiver that consists of a finite-complexity ML detector and a corresponding front-end equalizer. We will focus on multicarrier in this short conference paper, but note that the same equalizer is also optimum in a parallel sense for sequence detection in a more complete version of this paper.

The measure of complexity for an ML detector will be specified by ν, the order of a finite-length polynomial that is used in the design of the finite-complexity ML receiver. That is, both sequence detectors and multicarrier receivers need specification of some polynomial $b(D) = \sum_{m=0}^{\nu} b_m D^m$ that is used to approximate the bandlimited channel characteristic at some sampling rate (and D is the complex delay variable corresponding to that sampling rate) in the discrete-time approximation to the channel. The multicarrier receiver requires a block length N (basically twice the number of subcarriers) that is significantly larger than ν to implement the receiver without significant rate loss (a factor of $\nu/(N+\nu)$ in multicarrier implementations). The sequence detector requires M^{ν} states where M is the number of transmitted symbol levels in a single-carrier or baseband transmission method. In both ML receivers, receiver cost is dominated by elements that depend directly on ν so that a fixed cost of implementation will force a given maximum value of ν that can be tolerated in the design. With ν as the complexity constraint parameter, the function of an equalizer is then to filter the channel so that the combined equalizer-channel characteristic closely approximates a $b(D)$ that is chosen as the best $b(D)$ for the given channel. This concept is generically illustrated in Figure 1. For a given channel length of $\nu + 1$ samples, we need to find the best polynomial $b(D)$ and equal-

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izer so that the ML detector performance is optimized when designed for that $b(D)$. We will suggest that the best such choice of (nonzero) $b(D)$ minimizes the mean square error between the equalizer output and the filtered channel-input sequence $x(D)b(D)$ in Section 2.

Section 2 also introduces notation and discusses a general approach to decision-aided equalization, paralleling some earlier developments of [4], but continues on that earlier theory by focusing on only those receivers that can be implemented without need for any previous or future decisions. We derive in Section 2 a class of equalization structures that all have the same performance, and this class is found to include the MS-WMF that was conjectured in [5] to be a canonical receiver front-end, and the so-called Auto-Regressive Moving Average (ARMA)-based filter that was empirically found in [6] to be a good front-end for some multicarrier implementations. This class is also believed to include structures that are the infinite-length limits of some earlier systems derived by Zervos [7], [8] for M-ary single-carrier transmission on intersymbol interference channels. Our structures differ from those of Zervos in that the equalization filtering and the ML detection are separable, for instance permitting the use of the same equalizer with ML multicarrier demodulator or a sequence detector. This motivates our investigation of the case for finite ν in Section 3.

In Section 3, by constraining not only ν, but also the complexity of the equalizer, we find the optimum finite-impulse-response equalizer settings.[1] In this finite-length case, the relative delay characteristics of the equalizer and of $b(D)$ become crucial, especially as ν and the length of the equalizer become increasingly constrained. We introduce a search procedure that determines the best of the relative delay characteristics and quite often produces an equalizer that is not closely related to the feedforward filter of decision feedback, the latter of which might have been a designer's choice previous to the results of this paper.

With the results of Section 3, it is possible to analyze specific transmission channels and to ascertain the minimum ν and equalizer settings to approximate infinite-complexity performance closely. We often find the computation reduction is very large compared to the straightforward ML implementation and that the resulting reduced complexity is comparable (or often even less complex) to previous suboptimal equalization approaches. In Section 4, we consider an actual design example that is based on a study of digital subscriber loops. Significant complexity reduction occurs for negligible performance loss with respect to the optimum.

2 Infinite-Length Equalizers

The general baseband complex decision-aided, or in our study **input-aided**, equalization problem appeared in Figure 1. The channel output $\tilde{y}(t)$ is modelled as

$$\tilde{y}(t) = \sum_m x_m p(t - mT) + n(t) \qquad (1)$$

[1] We note this deviates from an earlier approach of Zervos [7], [8] in which only ν is fixed but not the complexity of the equalizer.

where x_k is the channel input **symbol**, supplied at some **symbol rate** $1/T$, $p(t)$ is the channel **pulse response** (the convolution of the **transmit filter**, $\phi(t)$, and the channel **impulse response** $h(t)$, $p(t) = \phi(t) * h(t)$), and $n(t)$ is additive white Gaussian noise (AWGN) with power spectral density $N_0/2$ per real dimension. The channel input symbols may not be M-ary symbols in the case of multicarrier modulation or coded single-carrier modulation, and in our development are only characterized as being independent identically distributed discrete-time samples with zero mean and **symbol energy** $E\{|x_k|^2\} = \mathcal{E}_x$. The channel output signal $\tilde{y}(t)$ is convolved with a matched filter $p^*(-t)$ (where a superscript of $*$ denotes conjugate), sampled at rate $1/T$, and then filtered by a linear equalizer. It is well-known [3] that matched-filter output is information lossless, and further ([9], Chapter 4) that when the equalizer is invertible that its output is also information lossless with respect to the channel input. This means that a maximum-likelihood detector applied to the signal at the output of the equalizer w_k in Figure 1 is equivalent in performance to the overall maximum-likelihood detector for the channel. We shall choose minimum mean-square error between the equalizer output and the desired channel shaping as a good measure of equalizer performance.

The **error sequence** can be written as $e_k = b_k * x_k - w_k * y_k$ where $*$ denotes convolution. The **signal** is defined as the sequence $b_k * x_k$, which has signal energy $\mathcal{E}_s = \|b\|^2 \mathcal{E}_x$. The **D-Transform** of a sequence x_k will be defined (see [5] for a more detailed development) by $x(D) \triangleq \sum_k x_k D^k$. We often use the notation $x^*(D^{-*}) = \sum_k x^*_{-k} D^{-k}$ to represent the time-reversed conjugate of the sequence. We can thus write

$$e(D) = b(D)x(D) - w(D)y(D) \quad . \qquad (2)$$

We minimize the **mean-square error** as $\sigma_e^2 = \min_{w_k, b_k} E\left[|e_k|^2\right]$ where E denotes statistical expectation. A trivial solution is $w(D) = b(D) = 0$ with $\sigma_e^2 = 0$. To avoid this trivial solution, we constrain the signal power to be positive so that $\mathcal{E}_s > 0$ or, equivalently, $\|b\|^2 = $ constant > 0. A related quantity is the **channel signal-to-noise ratio**

$$\mathrm{SNR}_{\mathrm{MFB}} = \frac{\mathcal{E}_s}{\sigma_e^2} \quad . \qquad (3)$$

When $\|b\|^2 > 0$, we will later see that $\mathrm{SNR}_{\mathrm{MFB}}$ is independent of $\|b\|^2$. Maximizing $\mathrm{SNR}_{\mathrm{MFB}}$ is then equivalent to minimizing σ_e^2. While our problem formulation is the same as the formulation of decision feedback equalization (see [5]), we differ here in that we do **not** restrict $b(D)$ to be causal or monic.

An **autocorrelation sequence** for any sequence x_k is defined by $r_{xx,k} \triangleq E\left[x_l x^*_{l-k}\right]$ with D-transform $R_{xx}(D)$. The **power spectral density** of the sequence is $R_{xx}(\theta) \triangleq R_{xx}(D)\,|_{D=e^{-j\theta}}$ and the energy of the sequence is $\mathcal{E}_x = r_{xx,0} = \frac{1}{2\pi}\int_{-\pi}^{\pi} R_{xx}(\theta)d\theta$. The **cross-correlation sequence** between two sequences x_k and y_k is then $r_{xy,k} = E\left[x_l y^*_{l-k}\right]$ with D-transform $R_{xy}(D)$. It will be convenient to write[2] $R_{xx}(D) = E\left[x(D)x^*(D^{-*})\right]$ and $R_{xy}(D) =$

[2] Actually, we are implying a limiting normalized sum for each coefficient in the resultant polynomial of the form $\lim_{L\to\infty} \frac{1}{2L+1}\sum_{k=-L}^{L}$.

$E\left[x(D)y^*(D^{-*})\right]$. As discussed in [5], any autocorrelation sequence that satisfies $\int_{-\pi}^{\pi} \ln|R_{zz}(\theta)|d\theta < \infty$ is canonically factorizable as $R_{zz}(D) = \mathcal{E}_z z(D) z^*(D^{-*})$ where $z(D)$ is monic ($z_0 = 1$), causal ($z_k = 0 \;\forall\; k < 0$), and minimum phase (all poles and zeros of the polynomial are outside the unit circle). The error sequence has autocorrelation transform $R_{ee}(D)$ and $r_{ee,0}$ is equal to the mean-square error.

A well-known principle in MMSE estimation is that the error sequence values e_k should be orthogonal to the inputs of estimation, some of which are the samples y_k. Thus $E\left[e_k y_l^*\right] = 0 \;\forall\; k, l$, which is compactly written

$$R_{ey}(D) = E\left[e(D)y^*(D^{-*})\right] = 0 \quad (4)$$

$$= b(D)R_{xy}(D) - w(D)R_{yy}(D) \quad . \quad (5)$$

then,

$$w(D) = b(D)\frac{R_{xy}(D)}{R_{yy}(D)} \quad . \quad (6)$$

We note that $y(D) = R_{pp}(D)x(D) + n(D)$, where $R_{pp}(D) \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_n |P\left(\frac{\theta - 2\pi n}{T}\right)|^2 d\theta$. Then $R_{xy}(D) = \mathcal{E}_x R_{pp}(D)$ and $R_{yy}(D) = R_{pp}(D)\left(\mathcal{E}_x R_{pp}(D) + N_0\right)$. We define the **channel autocorrelation** as $R_0(D) \triangleq \mathcal{E}_x R_{pp}(D) + N_0$, which always has canonical factorization (when $N_0 > 0$),

$$R_0(D) = \mathcal{E}_0 g(D) g^*(D^{-*}) \quad . \quad (7)$$

This factorization is called the "key equation" in [5].

We compute and simplify the error autocorrelation sequence as

$$R_{ee}(D) = \left(\frac{\mathcal{E}_x}{\mathcal{E}_0}\right) N_0 \cdot \frac{b(D)b^*(D^{-*})}{g(D)g^*(D^{-*})} \quad . \quad (8)$$

Defining the (squared) norm of a sequence as $||x||^2 \triangleq \sum_{k\to-\infty}^{\infty} |x_k|^2$, the mean square error from (8) is $\sigma_e^2 = \left(\frac{\mathcal{E}_x}{\mathcal{E}_0}\right) N_0 ||\frac{b}{g}||^2$. From the Cauchy-Schwarz lemma, we know $||b||^2 = ||\frac{b}{g} \cdot g||^2 < ||\frac{b}{g}||^2 ||g||^2$. Thus the MMSE is

$$\sigma_e^2 = \left(\frac{\mathcal{E}_x}{\mathcal{E}_0}\right) N_0 \frac{||b||^2}{||g||^2} \quad , \quad (9)$$

which can be achieved by a multitude of $b(D)$, only one of which is causal and monic.

The MMSE solution to our equalization problem is then achieved by any $b(D)$ that satisfies

$$b(D)b^*(D^{-*}) = \frac{||b||^2}{||g||^2} g(D) g^*(D^{-*}) \quad . \quad (10)$$

From (6), one finds $w(D)$ from $b(D)$ as

$$w(D) = \frac{\mathcal{E}_x}{\mathcal{E}_0} \frac{b(D)}{g(D)g^*(D^{-*})} \quad , \quad (11)$$

which is always invertible when $N_0 > 0$. The channel signal-to-noise ratio is

$$\mathrm{SNR}_{\mathrm{MFB}} = \frac{\mathcal{E}_0}{N_0} ||g||^2 \quad , \quad (12)$$

independent of $b(D)$ as promised earlier.

Note again that any invertible choice of $w(D)$ would still lead to an output for which ML detection is equivalent to $\tilde{y}(t)$. Forney ([3]) chose a filter that was equivalent to the limit as $N_0 \to 0$ of the causal choice of $w(D)$ in (11), while Ungerboeck ([10]) in his study of generalized ML receivers chose $w(D) = 1$. Both of these choices lead to ML receivers that are also optimum, but neither choice also minimizes mean-square-error. We shall see that a choice that also minimizes mean-square error can lead to much lower complexity of implementation for a given level of performance.

As mentioned earlier, there are many choices for $b(D)$. The Minimum-Mean-Square Error Decision Feedback Equalizer (MMSE-DFE), see for instance [5], chooses $b(D) = g(D)$ so that $b(D)$ is monic and causal. The Auto Regressive Moving Average (ARMA) Equalizer corresponds to the choice $b(D) = \frac{\mathcal{E}_0}{\mathcal{E}_x} g^*(D^{-*})$. In this case, $b(D)$ is noncausal and maximum phase (all roots inside the unit circle). We call this structure an ARMA equalizer because $w(D) = 1/g(D)$ is monic, causal and minimum phase and because we can write $y(D) = \frac{b(D)}{w(D)}x(D) + \frac{e(D)}{w(D)}$, often called an ARMA model in the field of digital signal processing.

Both the MMSE-DFE and the ARMA equalizer have the same performance (when ML detection is used). Essentially the causality/noncausality conditions of the feedforward filters and feedback filters have been interchanged. A multicarrier modulation method used with $w(D)$ equalization would perform the same for either choice of $w(D)$ (or any other optimum choice).

Thus, it would appear that since all structures are equivalent, we might as well continue with the use of the MMSE-DFE, since it is a well-known receiver structure. However, when finite-length filters are used, this can be a poor choice in attempting to maximize performance for a given complexity of implementation. As a trivial example, consider a channel that is maximum phase, the feedforward filter for a MMSE-DFE, in combination with the matched filter, will try to convert the channel to minimum-phase ([11]) at the output of $w(D)$, leading to a long feedforward section when (as in practice) the matched-filter and feedforward filter are implemented in combination in a fractionally spaced equalizer ([12]). On the other hand, the ARMA equalizer feedforward section (combined with matched filter) is essentially (modulo an anti-alias filter before sampling) one nonzero tap that adjusts gain. The opposite would be true for a minimum-phase channel.

In data transmission, especially on cables or over wireless transmission paths, the channel characteristic is almost guaranteed to be of mixed-phase if any reflections (multipath, bridge-taps, gauge-changes, slight imbalance of terminating impedances) exist. It is then natural (and correct) to infer that the best input-aided equalization problem is one that chooses $b(D)$ and $w(D)$ to be both mixed-phase also. The problem then becomes for a finite-length feedforward section ($w(D)$) of $M + 1$ taps and a finite-length channel model of $\nu + 1$ taps ($b(D)$), to find the best $b(D)$ and $w(D)$ for an ML detector designed for $b(D)$, the signal constellation, and additive white Gaussian noise (even if the noise is not quite Gaussian or white). This is the problem addressed in Section 3 and the solution is often neither decision feedback nor ARMA filtering.

3 Finite-Length Filters

Having established the possibility that certain mixed- (or maximum-) phase filterings are best for finite-length $w(D)$ and $b(D)$ under the assumed signal-to-noise measure of performance, we proceed in this section to derive the settings for $w(D)$ and $b(D)$. In so doing, we will absorb matched filtering in a finite-length fractionally spaced feedforward equalizer, as it would most likely be implemented in practice (even if the feedforward filter is approximated through symbol-spacing).

With fractionally spaced equalizer sampling at time instants $t = kT - \frac{iT}{l}, i = 0, \ldots, l-1$, equation (1) becomes

$$\underline{\tilde{y}}_k = \sum_m x_m \underline{p}_{k-m} + \underline{n}_k, \tag{13}$$

where $\underline{\tilde{y}}_k^T = [\tilde{y}(kT)\, \tilde{y}(kT - \frac{T}{l}) \ldots \tilde{y}(kT - \frac{l-1}{l}T)]$, $\underline{p}_k^T = [p(kT)\, p(kT - \frac{T}{l}) \ldots p(kT - \frac{l-1}{l}T)]$,
and $\underline{n}_k^T = [n(kT)\, n(kT - \frac{T}{l}) \ldots n(kT - \frac{l-1}{l}T)]$. We also assume that $p(t)$ extends only over a finite time interval $0 \le t \le LT$, and thus $\underline{p}_k = 0$ for $k < 0, k > L$. Then, for M successive l-tuples of samples of $\tilde{y}(t)$,

$$\tilde{\mathbf{y}}_k = \mathbf{P}\mathbf{x}_k + \mathbf{n}_k, \tag{14}$$

where

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} \underline{\tilde{y}}_k \\ \underline{\tilde{y}}_{k-1} \\ \vdots \\ \underline{\tilde{y}}_{k-M} \end{bmatrix}, \quad \mathbf{x}_k = \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-M-L} \end{bmatrix}, \quad \mathbf{n}_k = \begin{bmatrix} \underline{n}_k \\ \underline{n}_{k-1} \\ \vdots \\ \underline{n}_{k-M} \end{bmatrix}, \tag{15}$$

and

$$\mathbf{P} = \begin{bmatrix} \underline{p}_0 & \underline{p}_1 & \cdots & \underline{p}_L & 0 & \cdots & 0 \\ 0 & \underline{p}_0 & \underline{p}_1 & \cdots & \underline{p}_L & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \underline{p}_0 & \underline{p}_1 & \cdots & \underline{p}_L \end{bmatrix}. \tag{16}$$

We assume that the length of $w(D)$ is $(M+1)l$, $\mathbf{w} = [\underline{w}_0\ \underline{w}_1\ \underline{w}_2 \ldots \underline{w}_M]$, and the length of $b(D)$ is $\nu + 1$, $\mathbf{b} = [b_0\, b_1\, b_2 \ldots b_\nu]$. The error is

$$e_k = \mathbf{b}\mathbf{x}_{k-\Delta} - \mathbf{w}\tilde{\mathbf{y}}_k, \tag{17}$$

where Δ is an integer corresponding to delay (positive) or advance (negative), and $\mathbf{x}_{k-\Delta}^T = [x_{k-\Delta}\ x_{k-\Delta-1} \ \ldots \ x_{k-\Delta-\nu}]$.

Having decided on the values for M and ν, we proceed to the following $\nu + 1$ cases: setting $b_i = 1$ for each i in turn, $0 \le i \le \nu$. For each case, equation (17) now becomes

$$e_k = x_{k-\Delta-i} - \mathbf{v}\mathbf{u}_k, \tag{18}$$

where

$$\mathbf{u}_k = \begin{bmatrix} \tilde{\mathbf{y}}_k \\ \mathbf{x}_{(k-\Delta)|i} \end{bmatrix}, \quad \mathbf{v} = [\mathbf{w}\ \mathbf{b}_i], \tag{19}$$

$\mathbf{x}_{(k-\Delta)|i}^T = [x_{k-\Delta} \ \ldots \ x_{k-\Delta-i+1}\ x_{k-\Delta-i-1} \ \ldots \ x_{k-\Delta-\nu}]$, and $\mathbf{b}_i = [-b_0 \ldots - b_{i-1}\ - b_{i+1} \ldots - b_\nu]$.

To minimize MSE, we set $E[e_k \mathbf{u}_k^*] = 0$, which leads to the following solution:

$$\mathbf{v} = R_{x_{k-\Delta-i}\mathbf{u}} R_{\mathbf{uu}}^{-1}, \tag{20}$$

and

$$\sigma_{MSE,i}^2 = E[e_k e_k^*] = \mathcal{E}_x - \mathbf{v} R_{\mathbf{u}x_{k-\Delta-i}^*}. \tag{21}$$

In each case, Δ is varied to minimize MSE. Note that the case $b_0 = 1$ corresponds to the familiar DFE solution.

After executing the above algorithm for all ν cases, the solution corresponding to the smallest $\sigma_{MSE,i}^2$ is the optimum equalizer setting.

For multicarrier systems, the effect of finite FFT size, as well as the effect of cyclic prefix [6] (signal has cyclic prefix while noise does not) contributes to the overall achievable data rate. Although using the MMSE solution provides near optimal performance, other solutions using the above algorithms should also be checked and compared. Table 1 summarizes the algorithm for determining the best equalizer for multicarrier systems.

Step 1	Fix desired values for M and ν.
Step 2	Set i=0.
Step 3	Calculate the MSE and the taps setting according to equation (18) to (21), with various values for Δ.
Step 4	Pick 2 to 3 different Δs from step 3 corresponding to the smallest MSE and save those equalizer settings.
Step 5	Increment i by 1; repeat step 3 and 4 until i exceeds ν.
Step 6	Check the achievable data rate with each equalizer setting, and pick the one with the highest throughput.

Table 1: Algorithm for finding the best finite-length equalizer for multicarrier systems

4 Application to Multicarrier Systems

To demonstrate the significance of our new approach, we use the channel response of a 9-kft, 26-gauge copper wire inside the carrier serving area (CSA) loops that was studied extensively in [6]. The structure of the multicarrier system that we consider is also described in detail in [6]. We fix the sampling rate of our multicarrier system at 640 kHz, with a block length of 512. We assume the noise impairments are intersymbol interference (ISI) and additive white Gaussian noise (AWGN) with a power spectral density of -110 mW/Hz and an average input power of 10 mW. The bit error rate is set at 10^{-7} and a noise margin of 6 dB is assumed throughout this section. No coding is used. Computer simulation is used to generate the achievable throughput.

We pick several different values for the length of the feedforward equalizer $M+1$, and the length of the cyclic prefix, ν. The shorter the equalizer length, the less computational power required and thus the lower the cost. The shorter the cyclic prefix, the less significant the data loss caused by the extra samples transmitted. The results for $M = 11$ and

$\nu = 7$ based on the algorithm outlined in Table 1 are shown in Table 2. Each column in the table lists the MMSE for the optimal Δ, and the data rate for the multicarrier system using the taps setting derived from the algorithm. The achievable data rates for the multicarrier system do include the penalty caused by cyclic prefix.

i	0	1	2	3
σ^2_{MSE} with Optimal Δ ($\times 10^{-3}$)	4.37	2.99	2.78	2.62
Δ	10	8	3	7
Data Rate (in Mbps)	1.791	1.765	1.776	1.741
i	4	5	6	ν
σ^2_{MSE} with Optimal Δ ($\times 10^{-3}$)	2.19	2.37	2.49	2.92
Δ	7	5	3	3
Data Rate (in Mbps)	1.799	1.784	1.796	1.798

Table 2: Achievable data rate for multicarrier systems with $M = 11, \nu = 7$

The achievable data rates among various values of i do not exhibit significant differences. This phenomenon manifests the good tolerence of the variations of the feedforward equalizer for the multicarrier system, and also raises the need for checking performance with other equalizer settings in addition to the optimal solution. Table 3 shows similar results for different values of M and ν.

i	0	1	2	3	ν
σ^2_{MSE} with Optimal Δ ($\times 10^{-3}$)	10.77	7.63	6.57	10.68	15.99
Δ	6	4	3	4	2
Data Rate (in Mbps)	1.813	1.815	1.812	1.713	1.660

Table 3: Achievable data rate for multicarrier systems with $M = 3, \nu = 4$

The most important advantage of our new approach compared to [6] is the ability to reduce the number of feedforward taps and the number of cyclic prefixes with virtually no performance degradation, as evident from Table 2 and Table 3. The achievable data rate actually improves when we reduce the length of the equalizer by 8 and the cyclic prefix by 3. This translates to over 5 million instructions per second (MIPS) of savings in terms of computational power. Thus a multicarrier receiver can be designed that requires less than 15 MIPS.

To illustrate the high performance/cost ratio of our new approach, we consider the extreme case in which no feedforward equalizer is used in the multicarrier system. Table 4 summarizes the achievable data rate for different values of ν. With the optimal size cyclic prefix, the throughput decreases by about 6.2% and the receiver complexity decreases from 14.8 MIPS to 12.2 MIPS . While the above rate loss may not be significant because of the relatively large block length of 512, other systems that have stringent end-to-end delay requirement may restrict the maximum block length to 128, in which case the resulting performance degradation would be unacceptable.

ν	6	13	19	25	31
Data Rate (in Mbps)	1.362	1.590	1.680	1.702	1.692

Table 4: Achievable data rate for multicarrier systems with no feedforward equalizer

5 Conclusions

In this paper we develop a theory that shows the equivalence and the optimality of a class of equalizers for ideal (infinite complexity) maximum likelihood receivers. We also include an algorithm for searching the best equalizer when finite length constraints are imposed. We find that the complexity for the multicarrier receiver can be significantly reduced while maintaining near optimal performance. This is particularly crucial if small block length has to be used for some applications.

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