A WEIGHTED FREQUENCY-DOMAIN LEAST SQUARES APPROACH FOR EQUALIZATION IN DISCRETE MULTITONE SYSTEMS

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ABSTRACT

In Discrete Multitone (DMT) transceivers, a time domain equalizer (TEQ) is used to shorten the effective channel impulse response such that a shorter cyclic prefix can be used. In this paper, we pose the TEQ design problem completely in the frequency domain by defining a frequency-domain weighted least squares cost function. The definition of the frequency-domain TEQ design criterion, we show, allows for the introduction of a weighting function to control the spectral shape of the TEQ, and facilitates important extensions particularly useful for the DMT system. The TEQ can be used to suppress the noise and interference in the DMT system, a feature especially useful in the frequency division multiplexing based DMT system. Also, in the echo-cancellation based DMT system, the TEQ can be used to jointly shorten the echo response thus reducing the complexity of the echo canceler.

1. INTRODUCTION

Multi-carrier modulation (MCM) has been proposed for parallel communication in late 1950s [1] based on the concept of creating multiple orthogonal subchannels over which several data streams can be sent without intersymbol interference (ISI). This modulation scheme provides flexibility for adapting to different channel environments by adjusting the energy and constellation size of each carrier. One implementation of MCM is the Discrete Multitone (DMT) system which uses the discrete Fourier transform (DFT) for modulation [2], [3]. DMT has recently been chosen as the industry modulation standard for asymmetrical digital subscriber line (ADSL) modems [4] and is also a candidate modulation scheme for very-high-speed digital subscriber line (VDSL) systems [5].

In the DMT system, a cyclic prefix of length γ provides a guard time between transmitted symbols. If the channel response is of length $\gamma+1$ or shorter, DMT symbols can be transmitted free of ISI. Using large γ values to compensate for the length of the channel response, however, decreases the efficiency (introduces an overhead of $\gamma/(N+\gamma)$ where N is the DMT symbol length). A time domain equalizer (TEQ) to shorten the effective channel impulse response has been the most popular equalization approach for DMT systems [6] – [12]. This approach typically approximates the channel transfer function in DMT system by an autoregressive moving average (ARMA) model:

$$H(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{z^{-d}\sum_{i=0}^{L-1} b_i z^{-i}}{\sum_{i=0}^{M-1} a_i z^{-i}} \tag{1}$$

A TEQ whose transfer function is equal to $A(z^{-1})$ can be introduced at the receiver side, such that the cascade of channel $H(z^{-1})$ and TEQ approximates a sufficiently short target response $B(z^{-1})$. In Eqn. (1), L is the length of the target response that should be less than or equal to $\gamma+1$, M is the length of the TEQ filter, and d is the delay of the target response. Usually, the channel impulse response can not be perfectly shortened by the TEQ such that some residual energy of the shortened impulse response will lie outside the $\gamma+1$ consecutive taps with the highest total energy. We can use the shortening signal to noise ratio (SSNR) [9] to measure the shortening capability of the TEQ, which is defined as the ratio of the energy in the largest consecutive $\gamma+1$ taps to the energy in the remaining taps of the shortened response.

The TEQ design methods given in [7] – [10], use the time-domain squared error at the TEQ output as the design criterion and hence can not control the TEQ spectral shape easily. In [11], we pose the TEQ problem completely in the frequency domain. We introduce the weighted frequency-domain least squares (WFD-LS) method and derive the corresponding algorithm by minimizing a squared cost function defined in the frequency domain. In this paper, we show that by introducing a weighting function, we can effectively control the TEQ spectral shape and improve the overall performance of the DMT system. We compare the performance of WFD-LS algorithm with the time-domain optimal (TD-OP) and time-domain least squares (TD-LS) algorithms proposed in [9] and show that different weighting functions can lead to different TEQ spectral shapes for the WFD-LS algorithm. We also discuss two important extensions of our TEQ design approach to further improve the DMT system performance.

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2. WEIGHTED FREQUENCY-DOMAIN LEAST SQUARES ALGORITHM

During the initial training phase in a DMT transceiver, a pseudo-random sequence is transmitted repeatedly over the channel to form a periodic signal. Suppose $X(e^{-j\omega_k})$ and $Y(e^{-j\omega_k})$ are the DFTs of the training and the received signals, then the channel frequency response can be estimated as:

$$H(e^{-j\omega_k}) = \frac{Y(e^{-j\omega_k})}{X(e^{-j\omega_k})} \equiv \frac{B(e^{-j\omega_k})}{A(e^{-j\omega_k})}, \ k = 0, \ldots, N-1 \quad (2)$$

We would like to model the channel as the ratio of $B(e^{-j\omega_k})$ and $A(e^{-j\omega_k})$, where $A(e^{-j\omega_k})$ and $B(e^{-j\omega_k})$ are the DFTs of the TEQ and target response with delay d respectively. If there is no error in the modeling, we achieve:

$$B(e^{-j\omega_k})X(e^{-j\omega_k}) = A(e^{-j\omega_k})Y(e^{-j\omega_k}), \ k = 0, \ldots, N-1 \quad (3)$$

or

$$H(e^{-j\omega_k})A(e^{-j\omega_k}) = B(e^{-j\omega_k}), \ k = 0, \ldots, N-1 \quad (4)$$

The coefficients of the channel response $\mathbf{h}$, the TEQ $\mathbf{a}$ and the target response $\mathbf{b}$ are assumed to be real, which makes their frequency responses conjugate symmetric. Hence we can introduce the following frequency-domain least squares cost function for the TEQ design:

$$E_{ls}(\boldsymbol{\theta}) = \sum_{k=0}^{N/2} W(k)|H(e^{-j\omega_k})A(e^{-j\omega_k}) - B(e^{-j\omega_k})|^2 \quad (5)$$

where $\boldsymbol{\theta}^T = [a_1, a_2 \ldots, a_{M-1}, b_0, b_1 \ldots, b_{L-1}]$ is the vector of all the parameters of the ARMA model to be estimated (note that a_0 is assumed to be 1), N is the fast Fourier transform (FFT) length (hence is even) as well as the DMT symbol length, k is the index of the subchannel frequencies, and $W(k)$ is a non-negative weighting function which can be used to control the TEQ spectral shape.

If we differentiate $E_{ls}(\boldsymbol{\theta})$ with respect to parameter vector $\boldsymbol{\theta}$, and set the result to zero, we obtain:

$$\boldsymbol{\Phi}\boldsymbol{\theta} = \boldsymbol{\beta} \quad (6)$$

where

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{21} & \boldsymbol{\Phi}_{22} \end{bmatrix}$$

$$\boldsymbol{\Phi}_{11} = \begin{bmatrix} \mu_0 & \mu_1 & \cdots & \mu_{M-2} \\ \mu_1 & \mu_0 & \cdots & \mu_{M-3} \\ \vdots & & & \vdots \\ \mu_{M-2} & \mu_{M-3} & \cdots & \mu_0 \end{bmatrix},$$

$$\boldsymbol{\Phi}_{12} = \begin{bmatrix} -p_{d-1} & -p_d & \cdots & -p_{d+L-2} \\ -p_{d-2} & -p_{d-1} & \cdots & -p_{d+L-3} \\ \vdots & & & \vdots \\ -p_{d-M+1} & -p_{d-M+2} & \cdots & -p_{d+L-M} \end{bmatrix},$$

$$\boldsymbol{\Phi}_{21} = \boldsymbol{\Phi}_{12}^T,$$

$$\boldsymbol{\Phi}_{22} = \begin{bmatrix} \rho_0 & \rho_1 & \cdots & \rho_{L-1} \\ \rho_1 & \rho_0 & \cdots & \rho_{L-2} \\ \vdots & & & \vdots \\ \rho_{L-1} & \rho_{L-2} & \cdots & \rho_0 \end{bmatrix},$$

$$\boldsymbol{\beta} = \begin{array}{l} [-\mu_1 - \mu_2 \ldots - \mu_{M-1} \\ \quad p_d \ p_{d+1} \ \cdots \ p_{d+L-1}]^T, \end{array}$$

and

$$\mu_i = \sum_{k=0}^{N/2} |H(e^{-j\omega_k})|^2 \cos(i\omega_k)W(k),$$

$$p_i = \sum_{k=0}^{N/2} [R_k \cos(i\omega_k) - I_k \sin(i\omega_k)]W(k),$$

$$\rho_i = \sum_{k=0}^{N/2} \cos(i\omega_k)W(k),$$

$$R_k = \mathrm{Re}\{H(e^{-j\omega_k})\}, \quad I_k = \mathrm{Im}\{H(e^{-j\omega_k})\}.$$

Hence solution of the set of linear equations shown in Eqn. (6) yields the optimal parameter vector $\boldsymbol{\theta}$ such that the frequency-domain squared-error in Eqn. (5) is minimized. We call the resulting algorithm the weighted frequency-domain least squares (WFD-LS) algorithm. Note that an unweighted version of $E_{ls}(\boldsymbol{\theta})$ is used in [13] for a continuous-time system modeling.

3. EXTENSIONS OF WEIGHTED FREQUENCY-DOMAIN LEAST SQUARES ALGORITHM FOR DMT SYSTEMS

Because of the leakage caused by finite length FFT operation at the receiver, the narrow band noise and interference can be spread outside the band deteriorating the neighboring subchannel SNRs. In [14], Kerckhove and Spruyt proposed a constrained MSE criterion for the frequency division multiplexing (FDM) based DMT system to reduce the leakage of stopband noise and interference into the passband. We can extend this idea by considering suppression of all the noise and interference within the transmission band and incorporating these into our frequency-domain least squares cost function [12].

In the ADSL standard, two types of DMT systems are defined for full-duplex operation. One is the FDM-based DMT, where frequency bandwidth is split into upstream and downstream bands. The other version of the DMT system is called EC-based DMT, where the frequency bands for upstream and downstream data transmission are overlapped. Shortening the echo response can reduce the complexity of the echo canceler whose computational requirement is propotional to the length of the echo impulse response [15], [16]. We can incorporate the joint channel and echo responses shortening idea of [9] into our WFD-LS algorithm [11].

4. SIMULATION RESULTS

In this section, we compare the performance of the WFD-LS algorithm with the two methods given in [9]: time-domain optimal (TD-OP) and time-domain least squares (TD-LS) algorithms. Also, we investigate the effect of different weighting functions on the resultant TEQ spectral shapes and SSNR values of the shortened channel response.

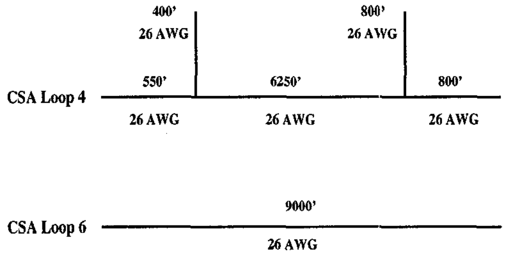

Figure 1: Test loop configurations

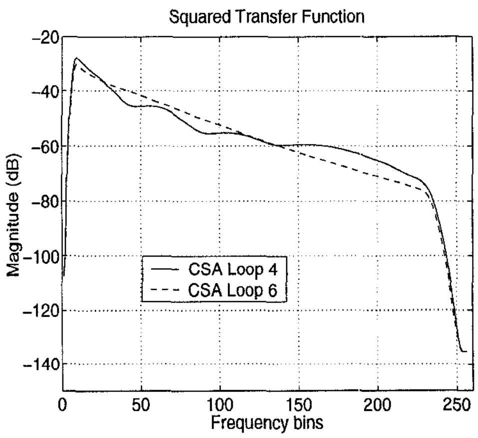

Figure 2: Channel transfer function

Figs. 1 shows the configurations for CSA loops 4 and 6. Their squared transfer functions when cascaded with a 30–1000 kHz bandpass filter are given in Fig. 2. In this simulation section, we only consider the downstream case, that is, $N = 512$ and $\gamma = 32$, and do not consider the additive noise effect in the channel.

First, two different weighting functions $W_a(k)$ and $W_b(k)$ shown in Fig. 3 are chosen for the WFD-LS algorithm. We apply three TEQ training algorithms (TD-OP, TD-LS and

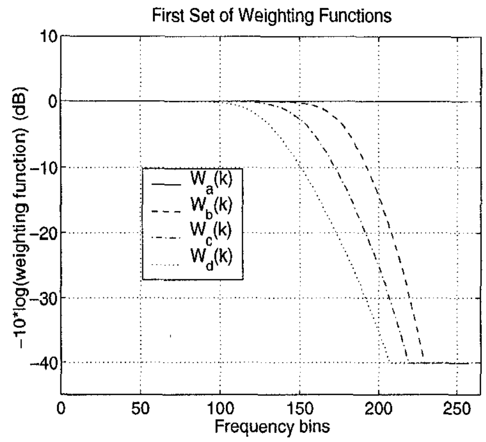

Figure 3: First set of weighting functions for WFD-LS algorithm

WFD-LS) to CSA loop 4 with TEQ length 20. The optimal delays of the target response for these four cases are determined in terms of maximum SSNR values. The second column of Table 1 shows the resultant SSNR values for the four cases for CSA loop 4. We can see that TD-OP algorithm achieves the best shortening performance because it explicitly works to maximize the SSNR [9]. The SSNR values obtained by TD-LS and WFD-LS with $W_a(k)$ weighting function are almost the same. For the WFD-LS algorithm with $W_b(k)$ weighting function, the resultant SSNR is less compared to that by TD-OP but better than those obtained by the other two.

Since the ultimate goal in the TEQ design is to achieve the maximum bit-rate, besides the SSNR measure, the TEQ spectral shape plays an important role for maximization of the channel throughput as it can be considered to be part of the overall channel response [14], [17]. Next, we discuss the resultant TEQ spectral shapes obtained by these three methods. We note that the TEQ spectral shape obtained by TD-OP results in deep nulls (see Fig. 4) although this method is optimal in terms of SSNR. All subchannels that coincide with these nulls or are near can only be assigned none or small number of bits, resulting in an overall frequency response that is undesirable [17]. The TEQ shapes obtained by TD-LS and WFD-LS with $W_a(k)$ weighting function for CSA loop 4 are very similar as shown in Fig. 4. When $W_b(k)$ is used as the weighting function for WFD-LS algorithm, the TEQ gain near the high frequency end is suppressed as would be expected by observing the characteristics shown in Fig. 3. We also apply these three methods to CSA loop 6 with TEQ length 14, and the results obtained are consistent with those obtained for CSA loop 4 as shown in Table 1 and Fig. 5.

Hence as shown in the simulations, WFD-LS allows for effective control of the TEQ shape by selecting proper weighting functions. The TD-OP algorithm is the most computationally complex one since it is based on eigenvector computation [9]. The computational requirements for the time-domain and frequency-domain LS methods on the other hand are similar because they both solve a set of linear equations to obtain the coefficients of the TEQ.

Table 1: SSNRs for different loops and algorithms

Channel	CSA loop 4	CSA loop 6
TD-OP	58.19 dB	64.87 dB
TD-LS	55.39 dB	61.07 dB
WFD-LS ($W_a(k)$)	55.37 dB	61.83 dB
WFD-LS ($W_b(k)$)	56.72 dB	63.08 dB

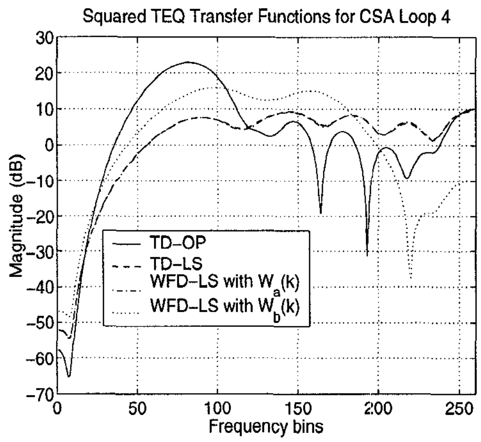

Figure 4: TEQ spectral shapes for CSA loop 4

Next, we show the relationship among different weighting functions, TEQ spectral shapes and the resultant SSNR values for WFD-LS algorithm. The TEQ length is chosen as 14 and the delay of the target response as 22 for all the following cases. We use the set of weighting functions shown in Figs. 3 and 7 for CSA loop 6. As seen in Figs. 6 and 8, the TEQ shape follows the weighting function trends in low or high frequency bins. What is more important to note is the change in the SSNR with different weighting functions. Higher suppression of the low frequencies leads to decrease in SSNR values and reverse for the high frequency bins. Hence, the results suggest higher contribution from the high frequencies to shortening noise and also reemphasize the fact that the SSNR, the index for shortening channel response, may not be the best measure for quantifying optimal performance.

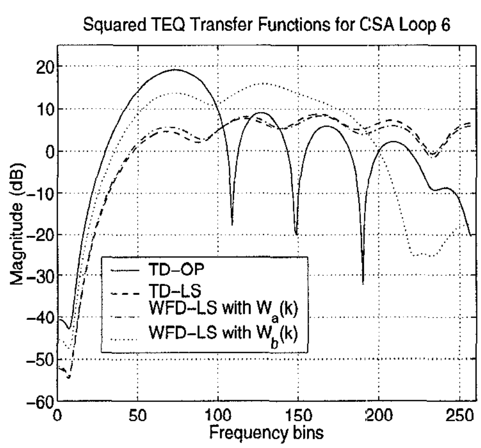

Figure 5: TEQ spectral shapes for CSA loop 6

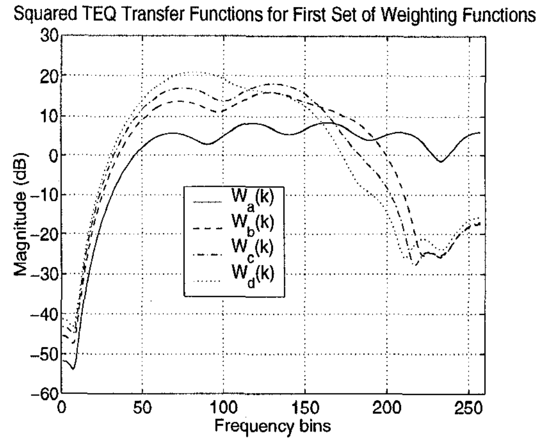

Figure 6: TEQ spectral shapes for the first set of weighting functions for CSA loop 6

Table 2: SSNRs obtained by WFD-LS algorithm for different weighting functions for CSA loop 6

Channel	SSNR (dB)
$W_a(k)$	61.83 dB
$W_b(k)$	63.08 dB
$W_c(k)$	63.44 dB
$W_d(k)$	64.03 dB
$W_e(k)$	60.95 dB
$W_f(k)$	58.99 dB
$W_g(k)$	57.34 dB

5. CONCLUSIONS

We introduce a frequency domain approach for designing the time domain equalizer for the DMT system. The least squares cost defined in the frequency domain allows for control of the TEQ magnitude response by a weighting function. The performance of the algorithm derived, WFD-LS, is comparable to that of time-domain least squares TEQ design when no weighting is used. As the simulation results suggest, by appropriate choice of the weighting function, the resulting TEQ shape can be controlled with the objective of improving the channel throughput. The TEQ shape also affects the SSNR values achieved, e.g., suppression of the higher frequency bands improving the SSNR, approaching the values obtained by the optimal time domain design approach, TD-OP, for the cases we studied. When noting the importance of the resulting TEQ shape on the overall DMT performance, it is also important to emphasize the presence of deep nulls in the spectrum achieved by the TD-OP leading to an undesirable overall TEQ response even though it is the best in terms of maximizing the SSNR, the objective function used in its design.

We also noted two other important extensions with the TEQ: echo and channel response shortening for the echo-

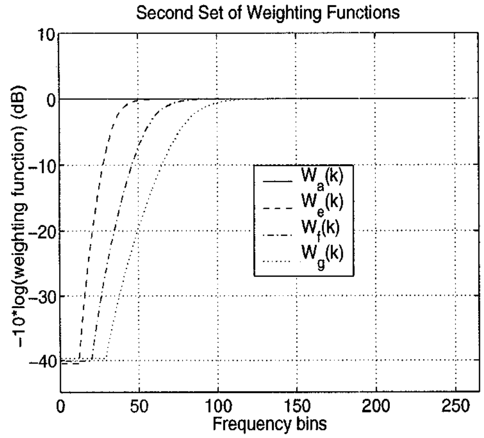

Figure 7: Second set of weighting functions for WFD-LS algorithm

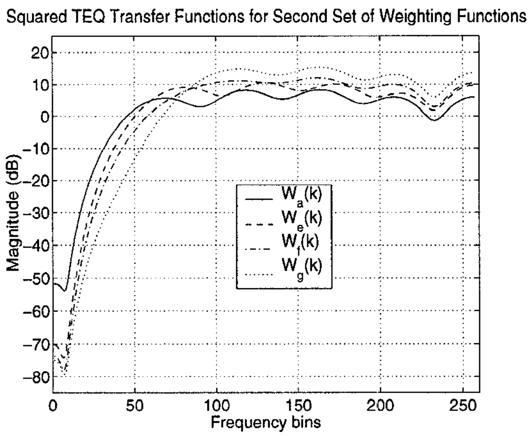

Figure 8: TEQ spectral shapes for the second set of weighting functions for CSA loop 6

cancellation based DMT systems for reducing the complexity of the echo canceler and joint noise suppression and channel response shortening for improving the overall DMT performance. For both cases, we can easily define the appropriate objectives directly in the frequency domain.

6. REFERENCES

[1] M. L. Doelz, E. T. Heald, and D. L. Martin, "Binary data transmission techniques for linear system," *Proc. IRE.*, vol. 45, pp. 656-661, May 1957.

[2] S. B. Weinstein, and P. M. Ebert, "Data transmission by frequency division multiplexing using the discrete fourier transform," *IEEE Trans. Communications*, vol. 19, no. 5, pp. 628-634, Oct. 1971.

[3] A. Peled, and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, pp. 964-967, (Denver, CO), Apr. 1980.

[4] J. A. C. Bingham, and F. van der Putten, T1. 413 Issue 2: Standards Project for Interfaces Relating to Carrier to Customer Connection of Asymmetrical Digital Subscriber Line (ADSL) Equipment, ANSI Document, No. T1E1.4/97-007R6, Sept. 1997.

[5] J. M. Cioffi, V. Oksman, J. J. Werner, T. Pollet, P. M. P. Spruyt, J. S. Chow and K. S. Jacobsen, "Very-high-speed digital subscriber lines," in *IEEE Communications Magazine*, pp. 72-79, Apr. 1999.

[6] J. S. Chow, J. M. Cioffi, and J. A. C. Bingham, "Equalizer training algorithms for multicarrier modulation systems," in *Proc. Int. Conf. on Communications*, pp. 761-765, (Geneva, Switzerland), May 1993.

[7] N. Al-Dhahir, and J. M. Cioffi, "Efficiently computed reduced-parameter input-aided MMSE equalizers for ML detection: A unified approach," *IEEE Trans. Inform. Theory*, vol. 42, no. 3, May 1996.

[8] M. Nafie, and A. Gather, "Time-domain equalizer training for ADSL," in *Proc. Int. Conf. on Communications*, pp. 1085-1089, (Montreal, Canada), June 1997.

[9] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, "Joint impulse response shortening for discrete multitone transceivers," *IEEE Trans. Communications*, vol. 44, no. 12, Dec. 1996.

[10] W. Chiu, T. C. Liau, W. K. Tsai, and M. Troulis "Time-domain channel equalizer design using the inverse power method," in *Proc. Int. Conf. on Communications*, (Vancouver, Canada), June 1999.

[11] B. Wang, and T. Adalı , "Joint impulse responses shortening for discrete multitone systems," in *Proc. IEEE Globecom Symposium on Communication Theory*, Rio de Janeiro, Brazil, Dec. 1999.

[12] B. Wang, and T. Adalı, "Time-domain equalizer design for discrete multitone systems," in *Proc. IEEE International Conference on Communications (ICC)*, New Orleans, LA, June 2000.

[13] E. C. Levy, "Complex-curve fitting," *IRE Trans. Automatic Control,*, vol. AC-4, pp.37-44, May 1959.

[14] J. V. Kerckhove, and P. Spruyt, "Adapted optimization criterion for FDM-based DMT-ADSL equalization," in *Proc. Int. Conf. on Communications*, pp. 1328-1334, (Dallas, TX), June 1996.

[15] M. Ho, *Multicarrier echo cancellation and multichannel equalization.* PhD thesis, Stanford University, June 1995.

[16] R. C. Younce, P. J. W. Melsa, and S. Kapoor, "Echo cancellation for asymmetrical digital subscriber lines," *Proc. Int. Conf. on Communications*, pp. 301-306, (New Orleans, LA), May 1994.

[17] B. Farhang-Boroujeny, and M. Ding, "An eigen-approach to the design of near-optimum time domain equalizer for DMT transceivers," in *Proc. Int. Conf. on Communications*, (Vancouver, Canada), June 1999.