

Design Methods for Time-Domain Equalizers in DMT Transceivers

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Abstract—Time-domain equalizers (TEQ) are used in the discrete multitone (DMT) transceivers in order to reduce the duration of the overall response of the transmission system, so that a shorter-length cyclic prefix could be used. The optimum TEQ is the one that results in maximum bit allocation to each block of DMT. However, the optimum design of TEQs turns out to be a very difficult task. In this paper, we give the general guidelines that one should follow in the design of TEQ to achieve a good performance. Based on the suggested guidelines, we first propose an eigenapproach design method which results in TEQs with comparable performance to those of a previously reported method, but at a much lower computational cost. Further study of the proposed guidelines reveals that the choice of target-impulse response in the design of TEQ only weakly depends on the channel response. Noting this, we propose a second design method that is even simpler than our first method, but still results in comparable designs to those of our first method and also those obtained from the much more complex methods of the present literature.

Index Terms—Data communications, discrete multitone, equalizers, subscriber loops.

I. INTRODUCTION

THE discrete multitone (DMT) has attracted considerable attention as a practical and viable technology for high-speed data transmission over spectrally shaped noisy channels [1]. Modems employing this technology are already available in the market. The DMT-based modems have, in particular, been found very useful in transmitting high-speed data over digital subscriber lines (DSLs). DMT is a special multicarrier data transmission technique that uses the properties of the discrete Fourier transform (DFT) in an elegant way so as to achieve a computationally efficient realization. Fig. 1 depicts a block diagram of a DMT modem. In the transmitter, the data sequence is partitioned into a number of parallel streams. Each stream of data is modulated via a particular subcarrier. The modulated subcarriers are summed to obtain the transmit signal. The use of DFT in DMT allows an efficient realization of the subcarrier modulators in a parallel processing structure which benefits from the computational efficiency of the fast

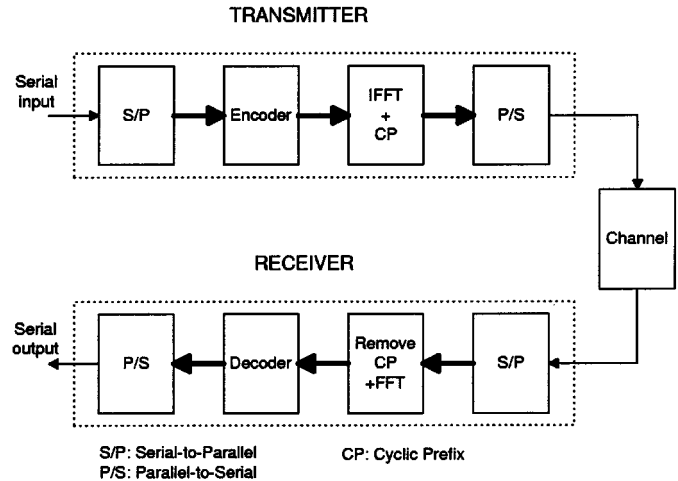


Fig. 1. Block diagram of a DMT transceiver.

Fourier transform (FFT). A similar DFT-based structure is used for efficient realization of the subcarrier demodulators in the receiver part of the DMT modem.

In DMT, channel distortion is taken care of by cyclically extending the output of the inverse FFT (IFFT) modulator so that the input sequence looks periodic to the channel. This is referred to as *cyclic prefix* method [1] (see Fig. 1). The length of the cyclic prefix should be at least equal to the duration of the channel impulse response L minus one. However, we note that the addition of the cyclic prefix reduces the throughput of the channel as it carries redundant data. To minimize this reduction of the throughput, a channel equalizer whose goal is to reduce the overall duration of the system (channel plus equalizer) impulse response to a predefined length is used. This accordingly allows us to use a shorter-length cyclic prefix. In the DMT literature, this type of equalizer is called a time-domain equalizer (TEQ).

The problem of TEQ design in the DMT transceivers may be formulated as follows. Given a channel with the impulse response samples h_0, h_1, \dots, h_{L-1} and corrupted with some additive noise, we wish to find the coefficients w_0, w_1, \dots, w_{N-1} of a transversal equalizer that results in a combined channel-equalizer response which is shortened to a duration of L_s samples, where L_s can be at most equal to the length of cyclic prefix plus one. In this design, the known parameters are the channel response, the channel noise (usually its autocorrelation coefficients), and the expected duration L_s of the equalized response. In the design process, we usually begin with the selection of a proper shortened impulse response known as target-impulse response (TIR). The TEQ coefficients are then selected so that the

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combined response of the channel and equalizer are as close as possible to the TIR. The criterion used for the selection of the TIR and TEQ may vary.

The ultimate goal in the design of the TEQ is to achieve maximum bit rate over the channel, given an acceptable level of bit-error probability. However, development of a practical design method that can achieve this goal turns out to be very difficult. Most of the studies in TEQ design have set the goal of mean-square-error (MSE) minimization which means the TIR and TEQ are jointly optimized so that the difference between the outputs of the TEQ and TIR is minimized in the MSE sense. This leads to an analytically tractable problem with a unique closed-form solution.

In the present literature, the only attempt that has been made in the design of TEQ with the goal of bit-rate maximization is the work of Al-Dhahir and Cioffi [3], [4], where the authors have proposed a rather complex optimization procedure. Convergence of this procedure to the corresponding optimum global solution is not guaranteed. However, numerical examples of actual digital subscriber line (DSL) channels have shown that the results obtained are superior to those of the MMSE TEQ.

In this paper, we consider a novel approach to the design of TEQ for DMT transceivers. Through numerical examples, we first explore the problem of TEQ design and propose some general guidelines that one should follow in order to arrive at a good design for TEQ. We show that the solution suggested by Al-Dhahir and Cioffi [3], [4] may be viewed as an attempt to satisfy these guidelines in a systematic way. We then propose an eigenapproach to the design of TEQ and through numerical examples demonstrate that this results in TEQ designs that are comparable to those of [3] and [4]; however, they are obtained with a much lower computational cost.

In our further study, we note that in the design of TEQ the optimum choice of the TIR weakly depends on the channel response. Noting this, we propose a second approach for the design of near-optimum TEQs. This approach involves classification of the class of channels into a few subclasses and use of a common TIR for all the channels in each subclass. This method which involves the use of some look-up tables is even simpler than our first method, thus more useful for practical implementation of the DMT transceivers in real time.

This paper is organized as follows. In Section II, we present the conventional method of TEQ design, known as MMSE TEQ. In Section III, through numerical examples, we discuss the shortcoming of MMSE TEQ in limiting the transmission bit rate. This will shed some light on the problem of TEQ design. In Section IV, we benefit from this understanding of TEQ and give some guidelines for the selection of TIR in the TEQ design. The optimization method proposed by Al-Dhahir and Cioffi [3] is reviewed in Section V where it is shown that this method results in a design which matches the guidelines set in Section IV. In Section VI, we present an eigenapproach to the design of TEQ. Some numerical examples that compare the results of the eigenapproach with those of [4] and also the MMSE TEQ are presented in Section VII. Our second design approach is presented in Section VIII. In Section IX, we present a statistical evaluation of this second approach by examining its performance over 5000 randomly generated

carrier-serving-area (CSA) loops. Concluding comments are made in Section X.

II. MMSE TEQ

As was noted earlier, in the MMSE TEQ the criterion used for the design is the MMSE at the equalizer output. The solution to this problem is well understood [2]. Let \mathbf{c} denote the length L_s column vector of the samples of the TIR. To prevent the trivial solution $\mathbf{c} = \mathbf{0}$, the unit-norm constraint is commonly applied to the TIR. That is, the variation of \mathbf{c} is limited by applying the constraint $\mathbf{c}^T \mathbf{c} = 1$, where the superscript T denotes transposition.¹ The optimum TIR, \mathbf{c}_{MMSE} , that results in the MMSE is then obtained according to the following optimization procedure:

$$\mathbf{c}_{\text{MMSE}} = \arg \min \{ \mathbf{c}^T \mathbf{R}_\Delta \mathbf{c} \} \quad (1)$$

subject to the constraint

$$\mathbf{c}_{\text{MMSE}}^T \mathbf{c}_{\text{MMSE}} = 1. \quad (2)$$

Here

$$\mathbf{R}_\Delta = \mathbf{I}_{L_s} - \mathbf{H}_\Delta^T \mathbf{R}^{-1} \mathbf{H}_\Delta \quad (3)$$

where \mathbf{R} is the $N \times N$ correlation matrix of the TEQ input, $\mathbf{H}_\Delta = \mathbf{H} \times [\mathbf{0}_{L_s \times \Delta} \quad \mathbf{I}_{L_s} \quad \mathbf{0}_{L_s \times L_r}]^T$ is an $N \times L_s$ matrix

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_{L-1} \end{bmatrix} \quad (4)$$

is an $N \times (N + L - 1)$ matrix, $\mathbf{0}_{m \times n}$ is the $m \times n$ null matrix, \mathbf{I}_m is the identity matrix of size m , Δ is the equalizer delay, and $L_r = N + L - \Delta - L_s - 1$. The solution to the above optimization is well understood [2], [6], [7]. The optimum solution, \mathbf{c}_{MMSE} , is the eigenvector that corresponds to the minimum eigenvalue of the matrix \mathbf{R}_Δ .

Once the optimum TIR is obtained, the TEQ coefficients can be calculated by solving the corresponding Wiener-Hopf equation or using an adaptive algorithm [2], [6], [7].

Also, for our later reference, we note that for a given TIR, \mathbf{c} , the MSE at the TIR output is given by

$$\xi = \mathbf{c}^T \mathbf{R}_\Delta \mathbf{c}. \quad (5)$$

III. SHORTCOMING OF MMSE TEQ

In this section, we demonstrate the shortcoming of the MMSE TEQ through a numerical example. The example given is a representative of many tests that we carried out over a large number of CSA loops. Consider loop 4 of the eight CSA loops whose configurations are presented in Fig. 5 and will be used for an extensive study/comparison of various TEQ design methods in Section VII. Figs. 2 and 3 present the magnitude

¹In this paper, with the exception of the matrix \mathbf{G}_i , which is introduced in Section V, we assume that all vectors and matrices are real-valued. When the involved vectors/matrices are complex-valued, all transpositions are replaced by Hermitian.

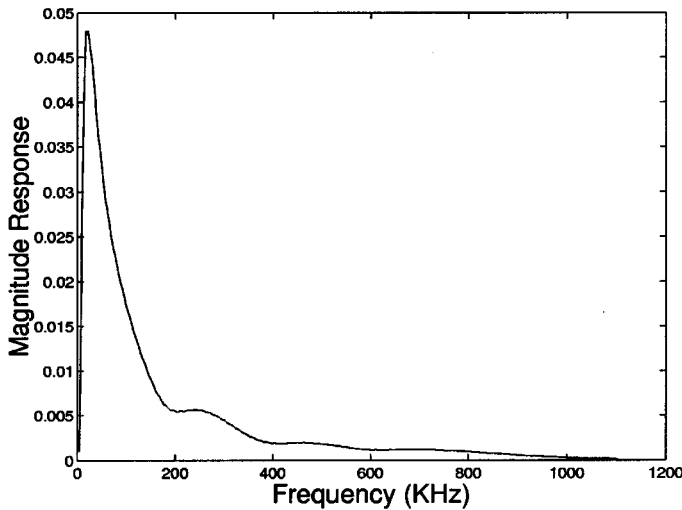


Fig. 2. Amplitude response of a typical ADSL channel (loop 4).

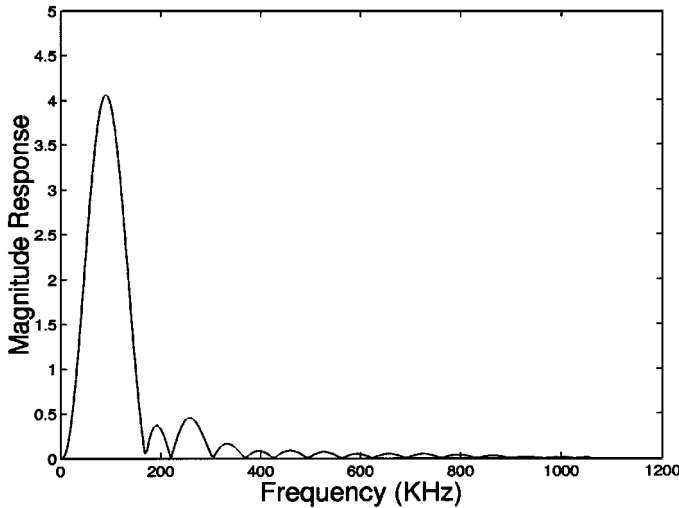


Fig. 3. Magnitude response of MMSE TEQ for loop 4.

responses of this loop and the corresponding MMSE TEQ, respectively. From these plots, we note that the MMSE TEQ has a very narrow band. It emphasizes only on the portion of the spectrum of the received signal that has the highest power. This observation may be interpreted as follows.

- Subject to the TIR duration and unit-length constraint, the MMSE TEQ does its best to pick up as much as possible of the less noisy part of the received signal, while suppressing the channel noise over the rest of the band as much as possible.

Although this maximizes the signal-to-noise ratio (SNR) of the received signal at the TEQ output and thus achieves the goal of the more conventional communication schemes, it does not necessarily result in the highest transmission bit rate in the DMT transceivers. To achieve the maximum bit rate in a DMT transceiver, one should strike a balance in the profile of the SNRs of all subcarriers after demodulation, so that a reasonably good number of bits can be allocated to most of the subcarriers, thus, overall, a large number of bits can be allocated to each block of DMT. As one may see by direct inspection of Fig. 3, the fact that the MMSE TEQ suppresses the received signal over most of its

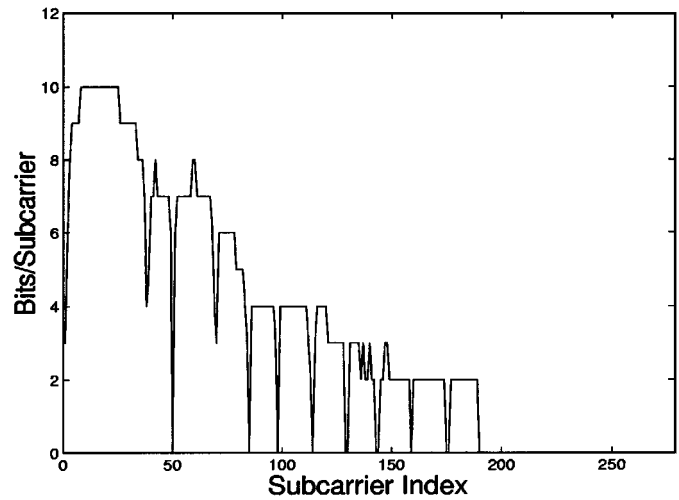


Fig. 4. Number of bits allocated to each subcarrier after MMSE TEQ in loop 4.

spectral band may result in a significant loss of information by lowering SNR in most of the subcarriers and, thus, an overall reduction in the total bits that could be allocated to each block of DMT. In particular, the subcarriers that may coincide with the nulls of the equalizer response will suffer a significant loss in their SNRs and, thus, the number of bits allocated to them.

To see this phenomenon clearly, Fig. 4 presents the bit allocation profile of the DMT transceiver for the above example. As was just predicted, we note that all subcarriers that coincide with the nulls of TEQ or are near the nulls have lower number of bits allocated to them. However, the subcarriers which are located near the middle of the side-lobes of the MMSE TEQ response still can support relatively fair numbers of bits, even though the peaks of these side-lobes are about an order of magnitude lower than the peak of the main-lobe of the response. This observation, which at first glance may seem to contradict our earlier arguments, needs some explanation.

Over each subcarrier band, the TEQ attenuates both the noise free part of the received (DMT) signal and the channel noise equally. As a result, if in the demodulation process one could use a bank of ideal narrow-band filters to separate different subcarriers, the receiver performance would be independent of the equalizer response. However, in a DMT system we use a DFT filter bank to separate the subcarriers. On the other hand, we recall that the DFT filters have relatively large side-lobes. As a result, there will be some leakage of noise energy from other bands to the main bands of the DFT filters. Those subcarriers whose amplitudes have been significantly attenuated (like those near the nulls of the TEQ) can thus be greatly affected by this noise leakage process. However, the effect of noise leakage to those bands whose amplitude attenuation is moderate, like those near the middle of the first or second side-lobes of TEQ, is insignificant.

IV. DESIGN GUIDELINES FOR GOOD TEQ DESIGN

From our discussion in the last section, we may conclude that to enhance the performance of the DMT transceivers, we shall choose a TEQ whose gain does not experience any null over the useful portion of the received signal band. We also note that the TEQ response is directly related to the TIR, c . In particular, the

nulls in the MMSE TEQ, which were observed in Fig. 3 and found to be undesirable, arise because of their presence in the amplitude response of the selected TIR, \mathbf{c}_{MMSE} . On the other hand, the MSE at the TEQ output varies with \mathbf{c} [see (5)]. With this understanding of the problem, we may now propose the following general guidelines for the design of TEQ in the DMT transceivers.

- The amplitude response associated with the TIR, \mathbf{c} , shall have no null.
- At the same time, \mathbf{c} should be chosen so that the MSE given by (5) remains relatively low.

The guidelines set above are relatively vague and seem to be hard to quantify. However, as we proceed in the rest of this paper, we find that the DMT transceivers are very robust to variations of the TIR, thus, many designs could be provided based on the above guidelines with all performing about the same.

V. AL-DHAHIR AND CIOFFI'S SOLUTION

In this section, we give an interpretation of the method of Al-Dhahir and Cioffi [3] and show how this method is related to the guidelines set in the last section.

To optimize the TIR, \mathbf{c} , Al-Dhahir and Cioffi [3] have proposed the following nonlinear optimization procedure:

$$\mathbf{c}_{\text{opt}} = \arg \max \left\{ \sum_i \ln(\mathbf{c}^T \mathbf{G}_i \mathbf{c}) \right\} \quad (6)$$

subject to the constraints

$$\mathbf{c}_{\text{opt}}^T \mathbf{c}_{\text{opt}} = 1 \quad (7)$$

and

$$\mathbf{c}_{\text{opt}}^T \mathbf{R}_{\Delta} \mathbf{c}_{\text{opt}} \leq \xi_{\text{max}} \quad (8)$$

where ξ_{max} is the maximum acceptable MSE at the TEQ output, and \mathbf{G}_i is an $L_s \times L_s$ matrix defined as

$$\mathbf{G}_i = \begin{bmatrix} 1 & e^{j2\pi i/N} & \dots & e^{j2\pi i(L_s-1)/N} \\ e^{-j2\pi i/N} & 1 & e^{j2\pi i/N} & \dots \\ \vdots & \ddots & \ddots & \vdots \\ e^{-j2\pi i(L_s-1)/N} & \dots & e^{-j2\pi i/N} & 1 \end{bmatrix} \quad (9)$$

This problem can be solved using an appropriate numerical software package. In [3] and [4] and also for the results presented in this paper, the MATLAB optimization toolbox is used. More specifically, the function “constr” of the toolbox is used [9]. This function performs the optimization by using a sequence of quadratic programming steps, where, at each step, the Hessian of the Lagrangian cost function associated with (6) and (7) is approximated using a quasi-Newton updating procedure. This is known as sequential quadratic programming in the literature [13], [14].

The above procedure suggests joint maximization of the terms $\ln(\mathbf{c}^T \mathbf{G}_i \mathbf{c})$, for $i = 1, 2, 3, \dots$, subject to the constraints (7) and (8). On the other hand, we note that the term $\mathbf{c}^T \mathbf{G}_i \mathbf{c}$ can be interpreted as the square of the gain of an FIR filter with tap-weight vector \mathbf{c} when its input is the complex sinusoid

$x_i(n) = e^{j2\pi i n/N}$, since \mathbf{G}_i is the autocorrelation function of $x_i(n)$. Thus, we say that Al-Dhahir and Cioffi's procedure makes an attempt to choose the samples of the TIR (i.e., the elements of \mathbf{c}) so that its gain values, over a dense grid of frequencies, defined by $\omega_i = 2\pi i/N$, $i = 1, 2, 3, \dots$, are maximized subject to the constraints (7) and (8). This procedure will converge to a TIR which has no null within the band of interest, since any null in the TIR response will force a few of the terms under the summation in (6) to some large negative values (because of the dense grid of frequencies). This clearly cannot be an optimal solution to the maximization procedure.

The above interpretation of the Al-Dhahir and Cioffi's solution matches nicely with the guidelines (design rules) that were set in the previous section. That is, subject to the unit-norm and MSE constraints set by (7) and (8), the optimum TIR, \mathbf{c}_{opt} , is found so that for all values of i , the terms $\mathbf{c}_{\text{opt}}^T \mathbf{G}_i \mathbf{c}_{\text{opt}}$ be nonzero. Because of dense grid of frequencies, this clearly is equivalent to preventing nulls in the TIR amplitude response.

VI. TEQ DESIGN BASED ON EIGENAPPROACH

In this section, we propose a new TEQ design scheme which uses the eigenvalues and eigenvectors of the matrix \mathbf{R}_{Δ} in order to select TIRs that satisfy the design guidelines of Section IV. The computational complexity of this scheme is much lower than the method of Al-Dhahir and Cioffi [3], [4], but the results of the two methods are comparable. As a measure of complexity, a TEQ design for the downlink of a typical ADSL channel by the method of Al-Dhahir and Cioffi takes more than 1 h on a SUN Ultra 30 Creator workstation, while the proposed scheme completes a similar design, on the same machine, almost instantly.

Let the column vectors $\mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_{L_s-1}$ and the scalars $\lambda_0, \lambda_1, \dots, \lambda_{L_s-1}$ be the unit-norm eigenvectors and the corresponding eigenvalues of \mathbf{R}_{Δ} , respectively. Then, according to the unitary similarity transformation [6], [7]

$$\mathbf{R}_{\Delta} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^T \quad (10)$$

where $\mathbf{Q} = [\mathbf{q}_0 \ \mathbf{q}_1 \ \dots \ \mathbf{q}_{L_s-1}]$ and $\mathbf{\Lambda}$ is a diagonal matrix consisting of $\lambda_0, \lambda_1, \dots, \lambda_{L_s-1}$. We also note that the eigenvectors $\mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_{L_s-1}$ are a set of mutually orthogonal vectors and, thus, may be considered as a set of bases vectors that can be used to express the vector \mathbf{c} as

$$\mathbf{c} = \sum_{i=0}^{L_s-1} \alpha_i \mathbf{q}_i = \mathbf{Q} \boldsymbol{\alpha} \quad (11)$$

where $\boldsymbol{\alpha} = [\alpha_0 \ \alpha_1 \ \dots \ \alpha_{L_s-1}]^T$ and $\alpha_i = \mathbf{q}_i^T \mathbf{c}$. A good choice of \mathbf{c} may thus equivalently be obtained by choosing a set of α_i 's that satisfy the guidelines of Section IV.

The term eigenfilter is often used to refer to FIR filters whose tap-weight vectors are the eigenvectors of a correlation matrix [6], [7]. Numerical examples show that the set of eigenfilters of \mathbf{R}_{Δ} happen to be a set of (almost) equally spaced narrow band filters. Any linear combination of these filters with some nonzero coefficients will thus result in a filter whose gain (with a good chance) remains nonzero over all frequencies. However, α_i 's cannot be chosen arbitrarily since they may result in a large undesirable MSE.

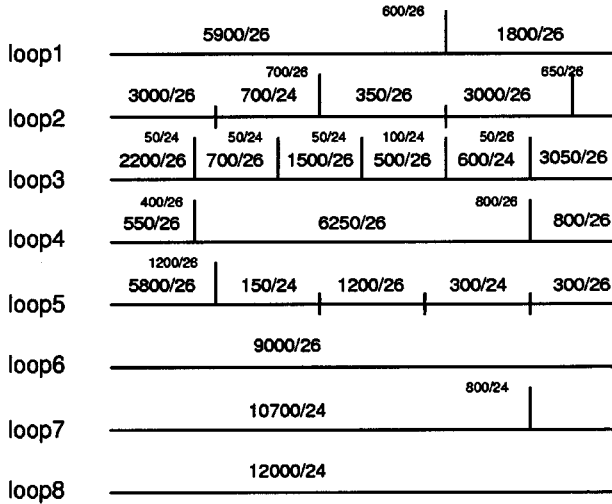


Fig. 5. Configurations of eight CSA loops.

Next, we suggest a simple *heuristic* choice of α_i 's that satisfies the guidelines of Section IV. To this end, we substitute (10) and (11) in (5), and rearrange the result, to obtain

$$\xi = \sum_{i=0}^{L_s-1} \alpha_i^2 \lambda_i. \quad (12)$$

We also note that the unit-norm constraint on \mathbf{c} implies that $\sum_{i=0}^{L_s-1} \alpha_i^2 = 1$. Hence, we may say the MSE of TEQ is given by a weighted average of the eigenvalues of \mathbf{R}_Δ . The weight factors are α_i^2 's. To keep the MSE of TEQ relatively low, we may simply choose α_i 's proportional to some negative power of λ_i 's. That is

$$\alpha_i = k \lambda_i^{-m}, \quad \text{for } i = 0, 1, \dots, L_s - 1 \quad (13)$$

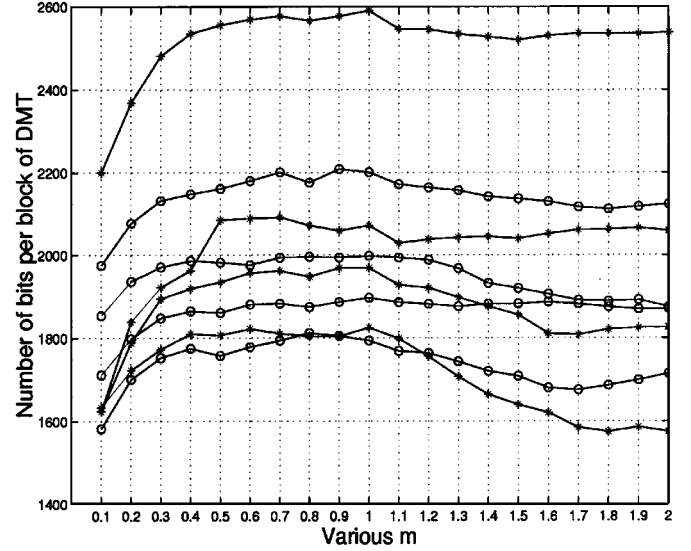
where k is a common factor which is chosen so that $\sum_i \alpha_i^2 = 1$. A good choice of the parameter m may be found experimentally (see the following section).

VII. SIMULATION RESULTS

To evaluate the performance of the proposed eigenapproach and also to compare it with the MMSE TEQ and the Al-Dhahir and Cioffi's method [3], [4], we present the results of a number of TEQs that we have designed for eight typical CSA loops. These loops, which are presented in Fig. 5, are those that have been considered in [3] and [4]. The DMT setup that we consider is the downlink of an ADSL transceiver. Accordingly, the IFFT and FFT lengths are 512, and a cyclic prefix length of 32 is assumed [10]. The signal power at the transmitter output is set equal to 14.1 dBm. We assume an additive white Gaussian noise (AWGN) with -110 dBm/Hz power and a near-end crosstalk (NEXT) whose power spectral density (PSD) is given by

$$S_{\text{NEXT}}(f) = k_{\text{NEXT}} f^{3/2} \times \frac{[\sin(\pi f/f_o) \sin(\pi f/2f_o)]^2}{[1 + (f/f_{1,3\text{dB}})^6] (f^2 + f_{2,3\text{dB}}^2)} \quad (14)$$

where $f_o = 1.455$ MHz, $f_{1,3\text{dB}} = 3$ MHz, $f_{2,3\text{dB}} = 40$ kHz, and $k_{\text{NEXT}} = 2.1581 \times 10^{-9}$. The NEXT PSD has been ob-

Fig. 6. Number of bits per each block of DMT as a function of parameter m .

tained from the ANSI T1.413-1998 standard [10]. The selected value of k_{NEXT} corresponds to 20 disturbers.

The bit allocation to different subcarriers is calculated according to the equation

$$b_i = \left\lfloor \log_2 \left(1 + \frac{\text{SNR}_i}{10} \right) \right\rfloor \quad (15)$$

where i varies over all data carrying subcarriers, SNR_i is the SNR at the i th subcarrier, and $\lfloor x \rfloor$ denotes the largest integer less than or equal to x . Equation (15) corresponds to bit-error probability of 10^{-7} for uncoded data. The total number of bits in each block of DMT is then $\sum_i b_i$, excluding the cases where $b_i = 1$. In the computation of the values of b_i , we have considered and used the bandwidth optimizing algorithm of [5]. That is, the subchannels that are unable to carry any information bit are taken note of and thus their corresponding signal powers are distributed among the data carrying channels. For more details on the bandwidth optimization algorithms, the reader may refer to [5] and [4] for their application to Al-Dhahir and Cioffi's scheme.

We performed an extensive set of simulations to find the best value of m to be used in (13). A summary of such simulations, for the CSA loops 1–8 of Fig. 5, are presented in Fig. 6. From these and other tests that we performed, it was found that $m = 1$, which also gives the simplest equation for computation of the TIR, is a good value. We also found that variation of m in the range of 0.5–1.5 has very little effect on the final total number of bits that will be allocated to each block of DMT. This robust behavior (or insensitivity) of the DMT to the choice of the parameter m , within some limited range, is attributed to the fact that the DMT is rather insensitive to the variations of the TIR. This is an interesting property of the DMT that may be exploited for further simplification of the TEQ design in DMT transceivers, as discussed in the next section.

Table I presents a summary of the results that we obtained for MMSE TEQ, the method of Al-Dhahir and Cioffi [4], and the proposed eigenapproach. From these results, we see that the eigenapproach method performs very similar to the method of

TABLE I
NUMBER OF BITS PER BLOCK OF DMT FOR VARIOUS
CSA LOOPS AND DIFFERENT DESIGNS

loop	MMSE TEQ	Al-Dhahir and Cioffi's [4]	Eigen-Approach
1	2266	2616	2590
2	1773	2087	2072
3	1597	1951	1969
4	1730	1914	1897
5	1953	2183	2201
6	1646	2015	1998
7	1639	1838	1825
8	1594	1789	1794

TABLE II
NUMBER OF BITS PER BLOCK OF DMT RESULTING FROM RANDOMLY
SELECTED TIRs, ACCORDING TO (16) FOR VARIOUS CSA LOOPS

loop	mean	standard deviation	maximum	minimum
1	2582	12.7	2621	2530
2	2049	12.8	2090	2004
3	1948	9.7	1974	1902
4	1900	9.4	1926	1865
5	2181	9.0	2204	2147
6	2002	9.1	2023	1968
7	1815	10.3	1844	1778
8	1777	8.8	1800	1726

[4]. Both methods succeed to perform better than the MMSE TEQ.

In our initial study of the TEQ design, we observed that although the TIR obtained by our method looked very different from the one obtained through the optimization method of [4] (or [3]), in both the time and, to some extent, frequency domain, there was almost no difference between the number of bits allocated to each block of DMT in the two designs. This interesting observation show that DMT transceivers are robust to some variation of the TIR. This observation is also found to be in line with our arguments in Section IV where the proposed guidelines were noted to be relatively vague. To explore this further, we generated 1000 different target responses according to the equation

$$\mathbf{c} = \left(\sum_{i=0}^{L_s-1} \frac{\beta_i}{\lambda_i} \mathbf{q}_i \right) / \sqrt{\sum_{i=0}^{L_s-1} \frac{1}{\lambda_i^2}} \quad (16)$$

where β_i 's are a set of independent random numbers taking values of +1 and -1. Each TIR was then used to obtain the corresponding TEQ and accordingly to calculate the number of bits allocated to each block of the DMT. Table II presents a summary of the results of this experiment that was repeated for all the eight loops of Fig. 5. In this table, we have given the mean of the number of bits allocated to each loop and their corresponding standard deviations, plus the maximum and minimum bit allocations that were observed in each experiment. The results clearly show the insensitivity of the DMT to these variations of the TIR. It is also instructive to compare these results with those of Table I and observe the following.

- 1) In all cases, on the average, the results obtained by the eigenapproach are superior to those of the MMSE design.

- 2) The results confirm the nonoptimality of the designs obtained by the method of Al-Dhahir and Cioffi [3], [4], as in all cases superior designs are found.
- 3) Most of the designs obtained through the TIR randomization perform about the same. The standard deviation of the number of bits allocated to each block in all cases remain within 1%–2% of the respective averages.

VIII. TEQ DESIGN BASED ON CHANNEL CLASSIFICATION

Our main observation in the previous sections was that in choosing TIR for TEQ design in DMT systems, we need only to comply with the design guidelines set in Section IV. Moreover, there are many choices of TIR that satisfy the design guidelines and thus result in TEQs that all perform well and about the same.

Another observation that we may have here is that the matrix \mathbf{R}_Δ , whose eigenvalues and eigenvectors could be used to select a well-behaved TIR according to (11) and (13), is directly related to the autocorrelation coefficients of the channel output. Furthermore, we note that the autocorrelation coefficients are related to the PSD of the underlying signal that, in turn, is directly related to the channel magnitude response and, for large SNRs, to a lesser extent to the noise PSD.

Considering these observations, in a recent patent [11] filed by the first author of this paper, a novel scheme has been proposed for design of TEQ in the DMT-based transceivers. In [11], only a general overview of the proposed scheme is given. Our aim in this section is to adopt the general scheme of [11] and give a detailed evaluation of that when applied to the particular case of ADSL channels.

The scheme proposed in [11] consists of two separate phases, namely, off-line and on-line.

The off-line phase consists of the following steps.

- 1) A large set of the members of the class of channels of interest are obtained, through measurements and/or simulations, whichever appropriate.
- 2) The channels with close responses are grouped together to make a number of subclasses.
- 3) The averaged responses of various subclasses are obtained and stored in a look-up table. These will be later used for identification of the subclass of the measured channel in the on-line phase of the design.
- 4) For each of the typical responses, a set of optimum (near-optimum) choices of the TIR and the delay parameter, Δ are obtained and stored in another look-up table.

The on-line phase of the proposed scheme consists of the following steps.

- 1) The channel response and the PSD of the channel noise are measured.²
- 2) The response of the measured channel is calculated and compared with the tabulated responses provided in Step 3 of the off-line phase, and accordingly the subclass of channel is identified, as explained below.

²In most of the application standards, including the ADSL standard [10], provision for such measurements during the system initialization is provided.

- 3) Once the subclass of the channel is identified, the corresponding TIR and delay parameter Δ are obtained from the look-up table generated in Step 4 of the off-line phase.
- 4) The TIR and delay parameter along with the channel response and the noise PSD are used to generate and solve the Wiener–Hopf equation leading to the equalizer coefficients.

The identification of the subclass of the measured channel should be done according to some *signature analysis*. Many choices of signatures are possible. The choice of the best signature is not trivial and may not be made in a very systematic way. However, fortunately, as was mentioned earlier, the channel magnitude response is closely related to the choice of a good TIR. Thus, a comparison of the measured channel magnitude response with the magnitude responses of the averaged channels obtained in Step 3 of the off-line phase is expected to give a good signature. This may be done as follows.

We assume that there are P subclasses. Let $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_P$ denote the column vectors representing the magnitude responses of the averaged channels obtained in Step 3 of the off-line phase. Also, let $\hat{\mathbf{a}}$ denote the column vector consisting of the samples of the magnitude response of the measured channel. We assume that the vectors $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_P$ and $\hat{\mathbf{a}}$ are of the same dimension, M . The closeness of $\hat{\mathbf{a}}$ to $\mathbf{a}_i, i = 1, 2, \dots, P$ is measured by minimizing the cost function

$$\gamma_i(\alpha) = \sum_{k=0}^{M-1} w(k)(\alpha \hat{a}(k) - a_i(k))^2 \quad (17)$$

where $\hat{a}(k)$ and $a_i(k)$ are the k th elements of $\hat{\mathbf{a}}$ and \mathbf{a}_i , respectively, and $w(k)$ is a weighting function. Through a large number of numerical tests on typical ADSL channels, we found that to get a better signature and thus a good TEQ, we should give a higher weight to the points where $\hat{a}(j)$ is relatively small. Considering this point and noting that for ADSL channels, generally, the magnitude response decreases over higher range of frequencies (see Fig. 2 as an example), we decided on a number of weighting functions that grow with frequency. After a number of trial and errors, we found the following weighting function as a good compromise choice for the CSA loops that we were experimenting on (see the following section for the details of the examined loops):

$$w(k) = \left(1 + \frac{k^2}{M^2}\right)^2. \quad (18)$$

Solving $d\gamma_i(\alpha)/d\alpha = 0$ for α , and using the result in (17), we obtain

$$\gamma_i^{\min} = \mathbf{a}_i^T \mathbf{W} \mathbf{a}_i - \frac{(\mathbf{a}_i^T \mathbf{W} \hat{\mathbf{a}})^2}{\hat{\mathbf{a}}^T \mathbf{W} \hat{\mathbf{a}}}, \quad \text{for } i = 1, 2, \dots, P \quad (19)$$

where \mathbf{W} is the diagonal matrix consisting of $w(0), w(1), \dots, w(M-1)$. The subclass of the measured channel is then determined by finding the index j that satisfies the following equation:

$$\gamma_j^{\min} = \min(\gamma_1^{\min}, \gamma_2^{\min}, \dots, \gamma_P^{\min}). \quad (20)$$

To have a fair comparison of these responses, the prestored vectors $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_P$ are all normalized to the unity norm. That is these have been normalized such that

$$\mathbf{a}_i^T \mathbf{a}_i = 1, \quad \text{for } i = 1, 2, \dots, P. \quad (21)$$

Although the above signature results in satisfactory TEQ designs for most of the channels, it fails to give good results for about 5% of the channels we tested. Through experiments, we found the use of a second signature, along with the above signature, could improve the results significantly. For the clarity of the discussions that follow, we refer to the signature introduced above as *signature 1* and the one introduced below as *signature 2*.

Numerical tests revealed that a good measure of distinguishing between channels that have similar amplitude responses, but their attachment to a particular subclass is not very strong, is the position of the peak of their time-domain impulse response. We quantify the peak positions, and thus implement the proposed signature 2, as follows. Suppose $\gamma_{j_1}^{\min}$ and $\gamma_{j_2}^{\min}$ are two extracts of signature 1 with the smallest and the second smallest value of all the γ_i 's, respectively. We form the following ratios:

$$R_1 = \frac{\min(\hat{\tau}, \tau_{j_1})}{\max(\hat{\tau}, \tau_{j_1})} \quad R_2 = \frac{\min(\hat{\tau}, \tau_{j_2})}{\max(\hat{\tau}, \tau_{j_2})} \quad (22)$$

and choose j_1 as the subclass of the measured channel if $R_1 \geq R_2$, and j_2 , otherwise.

IX. STATISTICAL EVALUATION OF TEQ DESIGN BASED ON CHANNEL CLASSIFICATION

In this section, we present a statistical evaluation of the TEQ design method of the previous section when it is applied to ADSL channels. For this, we start with a study of the responses of digital subscriber lines. Our study has shown that a choice of 20 subclasses gives a good variety of responses that match well with typical DSL loops studied in this paper. The choice of 20 subclasses has been based on a study of a wide range of channels that we generated in simulation using the channel parameters provided in the ADSL standard (T1.413-1998) for 24- and 26-gauge lines [10]. In generating the frequency responses of the channels, the two-port network equivalent of transmission lines, based on ABCD parameters, are used [8], [12].

The choice of 20 subclasses for classification of ADSL channels has also been based on other practical restrictions that are explained next. We note that, in general, the quality of the TEQ in terms of number of bits allocated to each block of DMT increases as the number of subclasses grows. On the other hand, the system complexity, both in terms of memory requirement (to store the magnitude responses of typical subclasses, plus the corresponding TIRs and delay parameters) and computational complexity (to do signature analysis and choose the best subclass) increases with the number of subclasses. Therefore, one should choose an appropriate number of subclasses that strikes a good balance between system complexity and its performance. The appropriate number that results in a good balance, in turn, depends on the available resources, i.e., DSP/hardware platform. The choice of 20 subclasses here, thus, has been somewhat

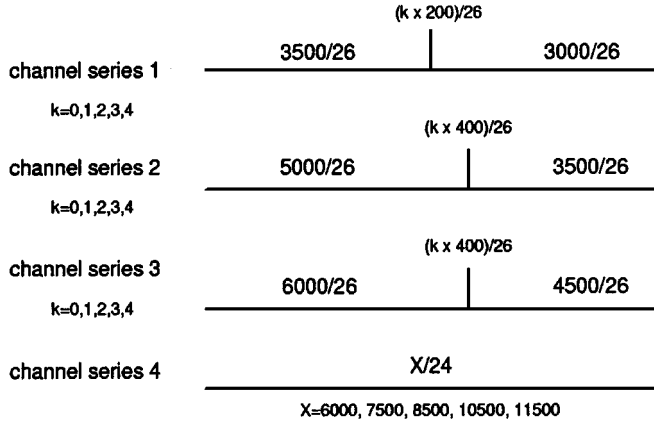


Fig. 7. The CSA loops chosen for channel classification.

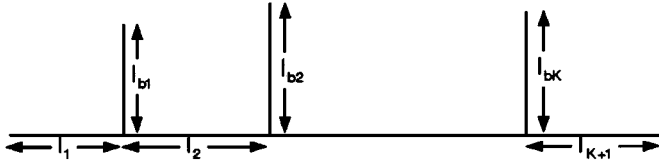


Fig. 8. The model of randomly generated CSA loops. The vertical lines are bridged taps. All lines are either 24 or 26 gauge.

arbitrary and may change depending on the available resources. However, it is an indicative figure.

Fig. 7 presents the details of the 20 loops that we consider as typical channels characterizing various subclasses. For each of these channels, we found the best TIR that we could get through a random search based on (16), with 1000 attempts for each loop. The TIRs obtained in this way and their corresponding delay parameters Δ , which were optimized by searching over all possible values that they could take, were then stored in a table. Clearly, all these operations correspond to the off-line phase of TEQ design.

To evaluate the on-line performance of the proposed design, we study the results of a large number of randomly generated CSA loops. Fig. 8 presents the details of the randomly generated loops. The parameters which are indicated in Fig. 8 are chosen randomly in the following order.

- *Principal line length, l* : a constantly distributed random variable in the range 5000–12 000 feet.
- *Number of bridged taps K* : takes values of 0, 1, 2, 3, or 4 with equal probability of occurrence.
- *Principal line segments, l_1, l_2, \dots, l_{K+1}* : a set of random variables with similar distribution that satisfy the following conditions:

$$l_1 + l_2 + \dots + l_{K+1} = l \quad (23)$$

$$l_1, l_2, \dots, l_{K+1} \geq l/10. \quad (24)$$

- *Length of the bridged tap loops, $l_{b1}, l_{b2}, \dots, l_{bK}$* : a set of random variables with constant distribution in the range of zero to one-tenth of the principal line length l .

All the lines are assumed to be 24 or 26 gauge. The PSD of the channel noise is assumed to be fixed and given according to the (14).

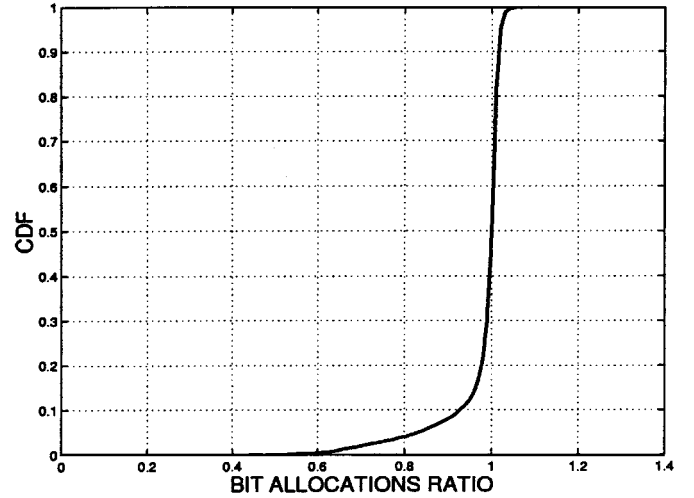


Fig. 9. CDF comparing the channel classification method against the eigenapproach. For channel classification, only signature 1 is used.

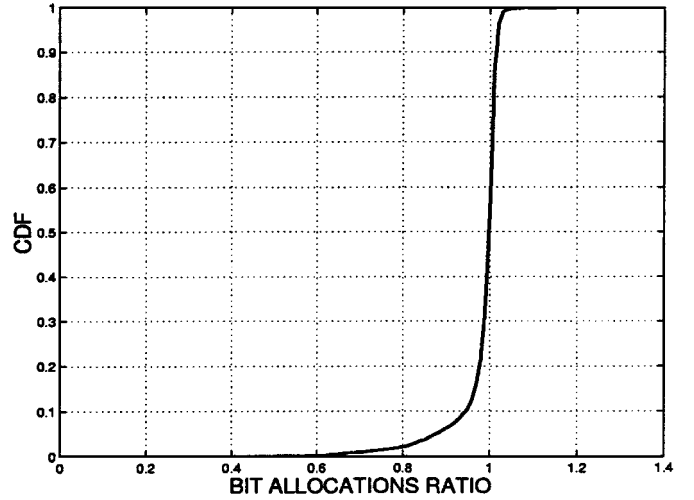


Fig. 10. CDF comparing the channel classification method against the eigenapproach. For channel classification, both signatures 1 and 2 are used.

For each loop, three TEQs are designed. The first TEQ is based on the eigenapproach method proposed in Section VI, with parameter $m = 1$. The other two are based on the channel classification method proposed in Section VIII, using the tabulated 20 subclasses mentioned above and the introduced signature analysis. From these, the first design uses only signature 1, while the second design uses both signatures 1 and 2. For each case, we calculate the maximum number of bits that could be allocated to each block of DMT based on error probability of 10^{-7} for uncoded data, subject to the maximum of 3000 bits per block (equivalent to a channel data rate of 12 Mb/s). Figs. 9 and 10 present summary of the results that obtained for 5000 randomly generated CSA loops.

Fig. 9 compares the performance of the channel classification method against the eigenapproach, when, for channel classification, only signature 1 is used. Fig. 10 presents the same results for the case where both signatures 1 and 2 are used to improve the results of the channel classification method. The horizontal axis shows the ratio of number of bits per DMT block (symbol) obtained from channel classification method over the

one obtained from eigenapproach. The vertical axis is the cumulative distribution function (cdf). The results clearly show that in a good majority of the cases (over 90%), the channel classification method and eigenapproach perform very similar. Moreover, by comparing Figs. 9 and 10, we find that the improvement brought about by adding signature 2 is only significant for the cases where the channel classification has badly performed. With signature 1 only (Fig. 9), in about 4% of the cases, the channel classification performance drops below 80% of what eigenapproach offers. While, with both signatures 1 and 2, the latter figure drops to about 2%. More improvement may be obtained if a better signature analysis could be found.

X. CONCLUSION

In this paper, we studied the problem of designing TEQs in the application of DMT transceivers. We noted that the optimal design of TEQ is a very difficult problem, and thus one has to resort to suboptimal solutions. To this end, based on an understanding of a shortcoming of the MMSE TEQ, we set some design guidelines for the selection of good TIR in TEQ design. We noted that the optimization method proposed in [3] is one possible method of achieving a design that satisfies the proposed guidelines. We also showed that there are other ways that could be used for selection of TIR that also satisfy the proposed guidelines and thus achieve good performance. In particular, we presented an eigenapproach and showed that it achieves similar performance to the designs obtained by the method of [3], however, at a much lower computational cost.

We also explored the possibility of storing a number of TIRs in a look-up table and choosing one of these TIRs for the design of TEQ, based on some signature analysis of the measured response of the channel. A statistical evaluation of this method showed that it works quite well for most of the simulated CSA loops and in the majority of the cases gives comparable results to those of the eigenapproach that was earlier found to be comparable with the method of [3] and [4]. This look-up table approach, which was referred to as channel classification method, is in particular a useful engineering approach to the design of TEQ in DMT transceiver in real time, as it greatly simplifies the design procedure.

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