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Per tone equalization for DMT-based transmission over IIR channels [1]

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[1]This report is available by anonymous ftp from *ftp.esat.kuleuven.ac.be* in the directory *pub/sista/vanbleu/reports/01-20.ps.gz*

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Per Tone Equalization for DMT-based Transmission over IIR Channels

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***Abstract*—Recently, an alternative receiver structure was presented for discrete multitone (DMT)-based systems. The traditional structure consisting of a (real) time domain equalizer (TEQ) with a (complex) 1-tap frequency domain equalizer (FEQ) per tone is modified into a structure with a (complex) multitap FEQ per tone. The signal-to-noise ratio (SNR) for each individual tone is maximized, hence the term "per tone equalization".**

Here, we derive an equalization scheme for DMT-based systems in an alternative way. We start from the assumption that the transmission channel to equalize has an infinite impulse response (IIR) or pole-zero model. We conclude that, under certain numerator and denominator conditions, the per tone equalizer is a close approximation of the optimal minimum mean square error (MMSE) equalizer. In case the numerator order condition is not fulfilled, we propose a low-complexity generalization of the per tone equalizer. This generalization is based on a suboptimal MMSE criterion and exploits transmit redundancy introduced by means of pilot and/or unused tones.

I. INTRODUCTION

Multicarrier modulation (MCM) has regained interest over the last decade. Several all-digital variants have been proposed: discrete multitone (DMT) is adopted as transmission scheme for asymmetric digital subscriber line (ADSL) and presented as a candidate for very high bit rate digital subscriber line (VDSL); orthogonal frequency division multiplexing (OFDM) is proposed for various wireless local area applications, e.g. HiperLAN Type 2 [1]. Throughout the text, we will use the term "DMT", although all principles are applicable to OFDM as well.

DMT schemes divide the available bandwidth into parallel subbands or tones. The incoming bitstream is split into parallel streams that are used to QAM-modulate the different tones. The modulation is done by means of an inverse fast Fourier transform (IFFT). Before transmission, a cyclic prefix is added to each symbol. If the channel impulse response order is less than or equal to the cyclic prefix length, demodulation can be implemented by means of an FFT, followed by a (complex) 1-tap frequency domain equalizer (FEQ) per tone to compensate for the channel amplitude and phase effects.

A long prefix however results in a large overhead with respect to the data rate. An existing solution for this problem is to insert a (real) T-tap time domain equalizer (TEQ) before demodulation, to shorten the channel impulse response. Many algorithms have been developed to initialize the TEQ (e.g. [2]). However a general disadvantage is that the TEQ equalizes all tones simultaneously and as a result limits the performance.

In [3], the authors propose a different receiver scheme that always results in better performance while keeping complexity during data transmission at the same level. Equalization is done for each tone separately after the FFT-demodulation, hence the term "per tone equalization".

The per tone equalizer is derived by transferring the TEQ operations to the frequency domain. In this paper, we propose an alternative derivation for the equalization of a DMT scheme. Starting point is the assumption that the transmission channel can be represented by an infinite impulse response (IIR) or pole-zero model. It will be shown that the per tone equalizer is a close approximation of the optimal minimum mean square error (MMSE) equalizer, on condition that the numerator order is less than or equal to the cyclic prefix length and the number of equalizer taps per tone is larger than the denominator order. The accuracy of the approximation depends on the colour of the noise and the denominator order. In addition, we present a generalization of the per tone equalizer, in case the numerator order condition is not fulfilled. The generalization is derived from a suboptimal MMSE criterion. It is based on the exploitation or addition of transmit redundancy by means of pilot tones and/or unused tones. Hence, by choosing the number of equalizer taps, the number of exploited pilot/unused tones and the cyclic prefix length, we implicitly fix an estimate of the IIR channel model order.

In section II, the data model and notation are introduced. Section III derives a per tone equalizer for three cases: (1) an IIR channel model with numerator order less than or equal to the cyclic prefix length, (2) an FIR channel model of arbitrary order and (3) the general case with arbitrary numerator and denominator order, which is a combination of case 1 and 2. Section IV evaluates the performance of the new, generalized per tone equalizer and studies the effect of the number of equalizer taps, the number of exploited pilot/unused tones and the cyclic prefix length in an ADSL context.

II. DATA MODEL AND NOTATION

In [5] and [6], it is stated that an IIR model offers a parsimonious representation of very long impulse responses in wired transmission applications such as DSL systems.

The IIR model $H(z)$ has a numerator $B(z)$ of order L_b and

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a denominator $A(z)$ of order L_a :

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{l=0}^{L_b} b_l z^{-l}}{\sum_{l=0}^{L_a} a_l z^{-l}}, \tag{1}$$

with $a_0 = 1$, which leads to the following relationship between transmitted samples $u[k]$, received samples $y[k]$ and noise samples $n[k]$:

$$\sum_{l=0}^{L_a} a_l (y[k-l] - n[k-l]) = \sum_{l=0}^{L_b} b_l u[k-l]. \tag{2}$$

In fact, the TEQ strategy is also based on the assumption of an IIR channel model: the TEQ should compensate for the denominator $A(z)$, while the 1-tap FEQ per tone cancels the effect of the numerator $B(z)$, provided $L_b \leq \nu$ (ν being the cyclic prefix length) [6].

Assume N is the (I)FFT size in transmitter and receiver, $s = N + \nu$ is the symbol block size (with prefix) and k is the block time index, then (2) gives rise to the following block-based description:

$$\underbrace{\begin{bmatrix} a_{L_a} & \cdots & a_0 & 0 & \cdots & 0 \\ 0 & a_{L_a} & \cdots & a_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & & \ddots & \\ 0 & \cdots & 0 & a_{L_a} & \cdots & a_0 \end{bmatrix}}_{\mathbf{A}_T} (\tilde{\mathbf{y}}_k - \tilde{\mathbf{n}}_k) =$$

$$\underbrace{\begin{bmatrix} b_{L_b} & \cdots & b_0 & 0 & \cdots & 0 \\ 0 & b_{L_b} & \cdots & b_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & & \ddots & \\ 0 & \cdots & 0 & b_{L_b} & \cdots & b_0 \end{bmatrix}}_{\mathbf{B}_T} \tilde{\mathbf{u}}_k \tag{3}$$

with $\mathbf{A}_T, \mathbf{B}_T$ Toeplitz matrices (size $N \times (N + L_b)$ and $N \times (N + L_a)$ respectively) and

$$\begin{aligned} \tilde{\mathbf{y}}_k &= \left[\ y[ks+\nu-L_a] \quad \cdots \quad y[(k+1)s-1]\ \right]^T \\ \mathbf{u}_k\tilde{\mathbf{n}}_k &= \left[\ n[ks+\nu-L_a] \quad \cdots \quad n[(k+1)s-1]\ \right]^T \\ \tilde{\mathbf{u}}_k &= \left[\ u[ks+\nu-L_b] \quad \cdots \quad u[(k+1)s-1]\ \right]^T. \end{aligned}$$

Since a linear convolution can be seen as a circular convolution with appropriate correction terms, we rewrite (3):

$$\mathbf{A}_C \mathbf{y}_k + \mathbf{A}_\Delta \Delta\mathbf{y}_k = \mathbf{B}_C \mathbf{u}_k + \mathbf{B}_\Delta \Delta\mathbf{u}_k + \mathbf{A}_T \tilde{\mathbf{n}}_k \tag{4}$$

with $\mathbf{A}_C$ and $\mathbf{B}_C$ circulant matrices (both size $N \times N$; $\mathbf{B}_C$ is readily obtained from $\mathbf{A}_C$):

$$\mathbf{A}_C = \begin{bmatrix} a_0 & 0 & \cdots & 0 & a_{L_a} & \cdots & a_1 \\ & \ddots & \ddots & & \ddots & \ddots & \\ a_{L_a-1} & \cdots & a_0 & 0 & \cdots & 0 & a_{L_a} \\ a_{L_a} & \cdots & a_1 & a_0 & 0 & \cdots & 0 \\ 0 & a_{L_a} & \cdots & a_1 & a_0 & \cdots & 0 \\ \vdots & & \ddots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{L_a} & \cdots & a_1 & a_0 \end{bmatrix},$$

with $\mathbf{A}_\Delta$ and $\mathbf{B}_\Delta$ tall Toeplitz matrices (size $N \times L_a$ and $N \times L_b$ respectively; $\mathbf{B}_\Delta$ is readily obtained from $\mathbf{A}_\Delta$):

$$\mathbf{A}_\Delta = \left[\begin{array}{cccc} a_{L_a} & a_{L_a-1} & \cdots & a_1 \\ 0 & a_{L_a} & \cdots & a_2 \\ \vdots & \ddots & \ddots & \\ 0 & \cdots & 0 & a_{L_a} \\ \hline \multicolumn{4}{c}{0_{(N-L_a)\times L_a}} \end{array} \right],$$

with the $N \times 1$ vectors

$$\begin{aligned} \mathbf{y}_k &= \left[\ y[ks+\nu] \quad \cdots \quad y[(k+1)s-1]\ \right]^T \\ \mathbf{u}_k &= \left[\ u[ks+\nu] \quad \cdots \quad u[(k+1)s-1]\ \right]^T \end{aligned}$$

and with the entries of the "difference terms" vectors in y and u,

$$\Delta\mathbf{y}_k = \left[\ \Delta y_k[1] \quad \cdots \quad \Delta y_k[L_a]\ \right]^T$$

and

$$\Delta\mathbf{u}_k = \left[\ \Delta u[1] \quad \cdots \quad \Delta u[L_b]\ \right]^T,$$

given by:

$$\begin{aligned} \Delta y_k[L_a + 1 - n] &= y[ks + \nu - n] - y[(k+1)s - n] \\ \Delta u_k[L_b + 1 - n] &= u[ks + \nu - n] - u[(k+1)s - n] \end{aligned}$$

with $n = 1, \cdots, L_a$ and L_b.

The vector $\mathbf{u}_k$ is the IFFT output, i.e. $\mathbf{u}_k = \mathcal{I}_N \mathbf{U}_k$ with $\mathcal{I}_N$ the $N \times N$ (unitary) inverse discrete Fourier transform (IDFT) matrix and $\mathbf{U}_k = \left[\ U_k[0] \quad \cdots \quad U_k[N-1]\ \right]^T$ the N transmitted complex subsymbols at block time index k. The receiver FFT output is $\mathbf{Y}_k = \mathcal{F}_N \mathbf{y}_k$ with $\mathcal{F}_N$ the (unitary) $N \times N$ DFT matrix.

To get to the final data model, we exploit the DFT-based decomposition of a circulant matrix, $\mathbf{X}_C = \mathcal{I}_N \mathbf{X}_D \mathcal{F}_N$, with $\mathbf{X}_D$ a diagonal matrix with the DFT of the first column of $\mathbf{X}_C$ as diagonal entries:

$$\mathbf{A}_D \mathbf{Y}_k + \mathcal{F}_N \mathbf{A}_\Delta \Delta\mathbf{y}_k = \mathbf{B}_D \mathbf{U}_k + \mathcal{F}_N \mathbf{B}_\Delta \Delta\mathbf{u}_k + \mathcal{F}_N \mathbf{A}_T \tilde{\mathbf{n}}_k. \tag{5}$$

In general, this is a set of N equations in $N + L_b$ unknowns $\mathbf{U}_k$ and $\Delta\mathbf{u}_k$. In practice however, some variables $U_k[n]$ and $\Delta u_k[n]$ are known at the receiver:

- $U_k[n] = 0$ for unused tones, e.g. in frequency division duplexing (FDD) ADSL, the unused tones $n = 30, \cdots, 37$ separate upstream and downstream;
- ν difference terms in u are zero because of the cyclic prefix:

$$\Delta u_k[L_b + 1 - n] = u[ks + \nu - n] - u[(k+1)s - n] = 0 \tag{6}$$

for $n = 1, \cdots, \nu$.

III. Per tone equalization over IIR channels

Based on (5), we will now derive a minimum mean square error (MMSE) equalizer for three cases. The first case assumes an IIR model with restricted numerator order $L_b \leq \nu$. The second case deals with an FIR model of arbitrary order, i.e. $L_a = 0, L_b > \nu$. Finally, we combine both cases yielding a solution for arbitrary L_a and L_b.

A. *Case 1: $L_b \leq \nu$*

If $L_b \leq \nu$, the difference terms in u vanish: $\Delta\mathbf{u}_k \equiv 0$. As a consequence, (5) is not underdetermined any more. The optimal MMSE estimate $\hat{\mathbf{U}}_k$ of $\mathbf{U}_k$ is given by:

$$\hat{\mathbf{U}}_k = \mathbf{R}_U\mathbf{B}_D^H \underbrace{\left(\mathbf{B}_D\mathbf{R}_U\mathbf{B}_D^H + \mathcal{F}_N \overbrace{\mathbf{A}_T\mathbf{R}_{\tilde{n}}\mathbf{A}_T^H}^{\mathbf{W}_T} \mathcal{I}_N\right)^{-1}}_{\mathbf{Z}} \times \left(\mathbf{A}_D\mathbf{Y}_k + \mathcal{F}_N\mathbf{A}_\Delta\Delta\mathbf{y}_k\right). \quad (7)$$

$(\cdot)^H$ denotes the Hermitian operation. $\mathbf{R}_U$ is the autocovariance matrix of the transmitted symbols $\mathbf{U}_k$. $\mathbf{R}_U$ is diagonal, assuming the tones are independently modulated, and has nonzero entries for all used tones. $\mathbf{R}_{\tilde{n}}$ is the noise autocovariance matrix.

The first term of $\mathbf{Z}$ in (7) is diagonal; the second term is not (unless $\mathbf{W}_T$ is a scaled identity matrix). However, $\mathbf{W}_T$ is a product of Toeplitz matrices, hence also Toeplitz. If only a band of m diagonals around the main diagonal of $\mathbf{W}_T$ is significantly different from zero, with $m \ll N$, $\mathbf{W}_T$ can be approximated quite well by a circulant matrix $\mathbf{W}_C$. It is said that $\mathbf{W}_T$ and $\mathbf{W}_C$ are asymptotically equivalent [7]. The smaller L_a and the less coloured the noise, the smaller m and the better the approximation[1]. Using the DFT-based decomposition of $\mathbf{W}_C = \mathcal{I}_N\mathbf{W}_D\mathcal{F}_N$, $\mathbf{Z}$ in (7) can be approximated by a diagonal matrix $\mathbf{Z}_D$:

$$\mathbf{Z}_D = \mathbf{B}_D\mathbf{R}_U\mathbf{B}_D^H + \mathbf{W}_D. \quad (8)$$

Provided $\mathbf{Z}_D$ is full rank, (7) reduces to an MMSE equalizer that is decoupled per tone. The symbol estimate $\hat{U}_k[n]$ of $U_k[n]$ only depends on the n-th instead of all N FFT outputs:

$$\begin{aligned}\hat{U}_k[n] &= B_D^*[n]\,\underbrace{\left(B_D[n]B_D^*[n] + W_D[n]\right)}_{Z_D[n]}{}^{-1} \times \\ &\quad \left(A_D[n]Y_k[n] + \mathcal{F}_N(n,:)\mathbf{A}_\Delta\Delta\mathbf{y}_k\right) \quad (9)\end{aligned}$$

where $(\cdot)^*$ denotes complex conjugate and every element of the form $(\cdot)_D[n]$ equals the n-th diagonal element of the corresponding diagonal matrix $(\cdot)_D$.

Equation (9) states that $\hat{U}_k[n]$ can be obtained as a linear combination of the n-th FFT output and L_a difference terms $\Delta\mathbf{y}_k$. Remarkably, this is the same result as was obtained in [3]. There, the authors started from the traditional TEQ to derive a receiver structure that optimizes the SNR per tone. The derivation above shows that the per tone equalizer is close to optimal (with accuracy depending on L_a and the noise colour) if $L_b \leq \nu$ and $L_a < T$ with T the number of equalizer taps per tone. The decoupled equalizer in (9) justifies a scheme with a moderate number of coefficients $T \ll N$ per tone that can be initialized recursively with training sequences and without prior knowledge of channel model and signal statistics [4].

B. *Case 2: $L_a = 0$, $L_b > \nu$*

In this case, $H(z)$ reduces to an FIR channel of order L_b. We assume that $H(z)$ has no zeros on the unit circle. The difference terms in y, $\Delta\mathbf{y}_k$, as well as the coefficients of $A(z)$ vanish. Equation (5) reduces to

$$\mathbf{Y}_k = \mathbf{B}_D\mathbf{U}_k + \mathcal{F}_N\mathbf{B}_\Delta\Delta\mathbf{u}_k + \mathcal{F}_N\tilde{\mathbf{n}}_k. \quad (10)$$

It follows from (6) that ν difference terms in u are zero, thanks to the cyclic prefix. The contribution of the nonzero difference terms in (10) is denoted by $\mathcal{F}_N\tilde{\mathbf{B}}_\Delta\Delta\tilde{\mathbf{u}}_k$. Still, we have N equations in $N + (L_b - \nu)$ unknowns. Extra transmit redundancy can be used to reduce the number of unknowns at the receiver. This redundancy must be added or is freely available: in a practical system, some tones are not used and/or there are pilot tones. We will refer to the set of unused and pilot tones as "nulltones" $\mathbf{U}_k^N$. Assume for simplicity that we use the minimal needed number of nulltones, i.e. $L_b - \nu$. The contribution in (10) of pilot tones (P) and used tones (U) is defined as:

$$\mathbf{B}_D\mathbf{U}_k = \mathbf{B}_D^P\mathbf{U}_k^P + \mathbf{B}_D^U\mathbf{U}_k^U \quad (11)$$

where $\mathbf{U}_k^P$ and $\mathbf{U}_k^U$ are column vectors with the pilot and used subsymbols respectively; $\mathbf{B}_D^P$ and $\mathbf{B}_D^U$ contain the corresponding columns of $\mathbf{B}_D$. Equation (10) can now be rewritten with all unknowns on the right-hand side:

$$\mathbf{Y}_k - \mathbf{B}_D^P\mathbf{U}_k^P = \underbrace{\begin{bmatrix} \mathbf{B}_D^U & \mathcal{F}_N\tilde{\mathbf{B}}_\Delta \end{bmatrix}}_{\overline{\mathbf{B}}} \underbrace{\begin{bmatrix} \mathbf{U}_k^U \\ \Delta\tilde{\mathbf{u}}_k \end{bmatrix}}_{\overline{\mathbf{U}}_k} + \mathcal{F}_N\tilde{\mathbf{n}}_k. \quad (12)$$

The optimal MMSE per tone equalizer for a system with sufficient transmit redundancy is then given by:

$$\begin{aligned}\hat{U}_k[n] &= \mathbf{R}_{\overline{U}}(n,:)\overline{\mathbf{B}}^H\left(\overline{\mathbf{B}}\mathbf{R}_{\overline{U}}\overline{\mathbf{B}}^H + \mathcal{F}_N\mathbf{R}_{\tilde{n}}\mathcal{I}_N\right)^{-1} \\ &\quad \times \left(\mathbf{Y}_k - \mathbf{B}_D^P\mathbf{U}_k^P\right) \quad (13)\end{aligned}$$

where $\mathbf{R}_{\overline{U}}$ is the autocovariance matrix of $\overline{U}_k$. This equalizer does not allow a decoupling per tone in the sense of (9). However, the zero forcing (ZF) equalizer for (12) does. Based on the following 2-step ZF equalizer design (assume no noise is present), we will obtain a suboptimal MMSE equalizer.

1. If $L_b > \nu$, $\mathbf{B}_D^U$ is a tall matrix with a left nullspace spanned by the columns of $\mathbf{B}_D^N$, i.e. the columns of the diagonal matrix $\mathbf{B}_D$, corresponding to the $L_b - \nu$ nulltones. After left multiplication of (12) with $\mathbf{B}_D^{N^H}$, $\Delta\tilde{\mathbf{u}}_k$ is given by:

$$\Delta\tilde{\mathbf{u}}_k = \left(\mathbf{B}_D^{N^H}\mathcal{F}_N\tilde{\mathbf{B}}_\Delta\right)^{-1}\mathbf{B}_D^{N^H}\left(\mathbf{Y}_k - \mathbf{B}_D^P\mathbf{U}_k^P\right). \quad (14)$$

[1] Note that in case of an FIR channel with an order less than or equal to ν and coloured noise, the same circulant approximation of the noise autocovariance matrix is needed to justify the use of a 1-tap FEQ per tone for the MMSE equalizer.

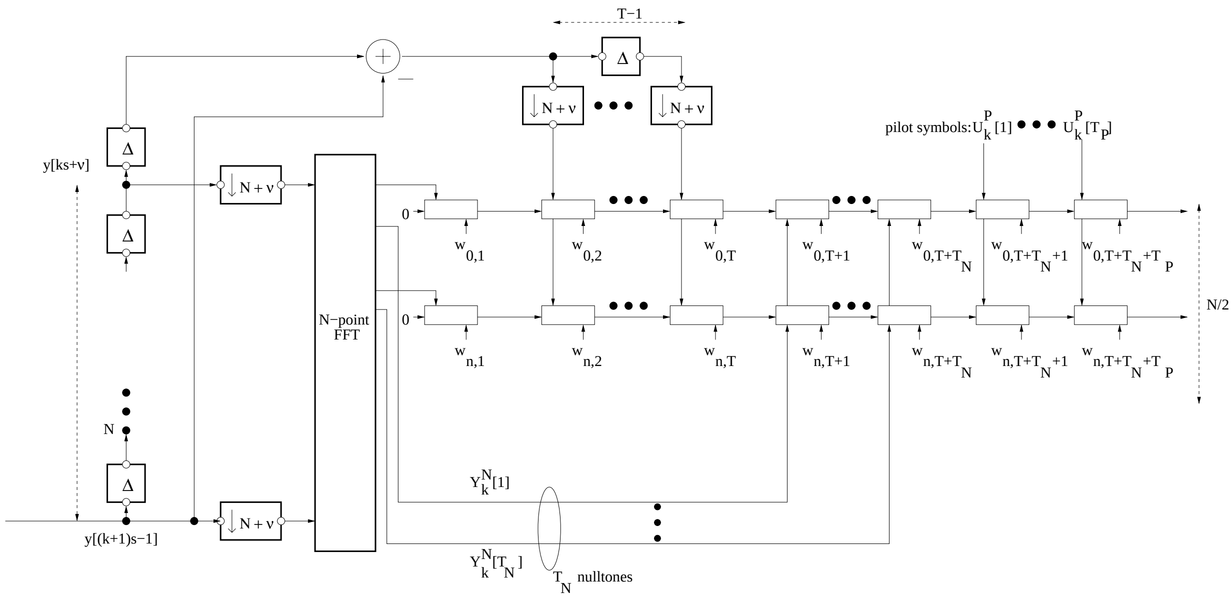

Fig. 1. The generalized per tone equalizer.

Because of the sparse structure of $\mathbf{B}_D^{N^H}$, $\Delta\tilde{\mathbf{u}}_k$ is a linear combination of $\mathbf{Y}_k^N$ (i.e. the FFT outputs $Y_k[n]$, with n corresponding to the set of nulltones $\mathbf{U}_k^N$) and the pilot symbols $\mathbf{U}_k^P$.
2. Eliminating the contribution of $\Delta\tilde{\mathbf{u}}_k$ in (12), $U_k[n]$ can be recovered as

$$\begin{aligned} U_k[n] &= \frac{1}{B_D[n]}\left(Y_k[n] - \Delta\tilde{\mathbf{u}}_k^T \mathbf{w}_{1,n}\right) \\ &= \frac{1}{B_D[n]}\left(Y_k[n] - \left[\begin{array}{cc} \mathbf{Y}_k^{N^T} & \mathbf{U}_k^{P^T} \end{array}\right] \mathbf{w}_{2,n}\right) \end{aligned} \quad (15)$$

where $\mathbf{w}_{1,n}$ and $\mathbf{w}_{2,n}$ are tone-dependent equalizer coefficients.
The suboptimal MMSE equalizer, based on this ZF equalizer, gives an MMSE estimate of $\Delta\tilde{\mathbf{u}}_k$ in a first step and uses this estimate to obtain $\hat{U}_k[n]$ in the second step. The overall result is[2]

$$\hat{U}_k[n] = \left[\begin{array}{ccc} Y_k[n] & \mathbf{Y}_k^{N^T} & \mathbf{U}_k^{P^T} \end{array}\right] \mathbf{w}_{3,n} \quad (16)$$

where $\mathbf{w}_{3,n}$ contains the equalizer coefficients for tone n and combines the n-th FFT output, the nulltone FFT outputs and the pilot subsymbols. It is clear that the performance of this suboptimal MMSE equalizer, compared to the optimal equalizer (13), improves with the SNR on the nulltones.

C. Case 3: arbitrary L_a and L_b

The general case with arbitrary L_a and L_b is the combination of Case 1 and 2. A suboptimal MMSE equalizer can thus be obtained by combining (9) and (16).
1. An MMSE estimate $\Delta\hat{\mathbf{u}}_k$ of $\Delta\tilde{\mathbf{u}}_k$ is obtained as a linear combination of the nulltone FFT outputs, $\mathbf{Y}_k^N$, the difference terms in y, $\Delta\mathbf{y}_k$, and the pilot subsymbols, $\mathbf{U}_k^P$:

$$\Delta\hat{\mathbf{u}}_k = \left[\begin{array}{ccc} \mathbf{Y}_k^{N^T} & \Delta\mathbf{y}_k^T & \mathbf{U}_k^{P^T} \end{array}\right] \mathbf{w}_{4,n} \quad (17)$$

with $\mathbf{w}_{4,n}$ the equalizer coefficients for tone n.
2. The MMSE symbol estimate $\hat{U}_k[n]$ is then given by:

$$\begin{aligned} \hat{U}_k[n] &= \left[\begin{array}{ccc} Y_k[n] & \Delta\mathbf{y}_k^T & \Delta\hat{\mathbf{u}}_k^T \end{array}\right] \mathbf{w}_{5,n} && (18) \\ &= \left[\begin{array}{cccc} Y_k[n] & \mathbf{Y}_k^{N^T} & \Delta\mathbf{y}_k^T & \mathbf{U}_k^{P^T} \end{array}\right] \mathbf{w}_n && (19) \end{aligned}$$

where the last transition comes from (17). $\mathbf{w}_{5,n}$ and $\mathbf{w}_n$ are the equalizer coefficients per tone.
Equation (19) suggests to extend the vector with difference terms $\Delta\mathbf{y}_k$ (which is common for all tones) in the per tone equalizer of [3] with T_N nulltone FFT outputs $\mathbf{Y}_k^N$ and T_P pilot subsymbols $\mathbf{U}_k^P$. The new structure is depicted in Fig. 1.

This equalizer can be initialized recursively without prior knowledge of channel and signal statistics, similar to [4]. By choosing a number of difference terms in y $(T-1)$, the number of nulltones (T_N) and a cyclic prefix length (ν), the equalizer identifies implicitly an IIR model with $L_a = T-1$ and $L_b = \nu + T_N$.

IV. Simulation results

The conclusion of the previous section suggests that one can trade-off the number of difference terms in y $(T-1)$, the number of nulltones (T_N) and the cyclic prefix length (ν). We check the influence of these parameters in an ADSL context. Simulations for several channel and noise configurations have been done with consistent results. Here, we depict the performance for an inline downstream channel of 4 km with additive white Gaussian noise. Four equalizer configurations are

[2] For the sake of conciseness, the detailed MMSE formulas have been omitted.

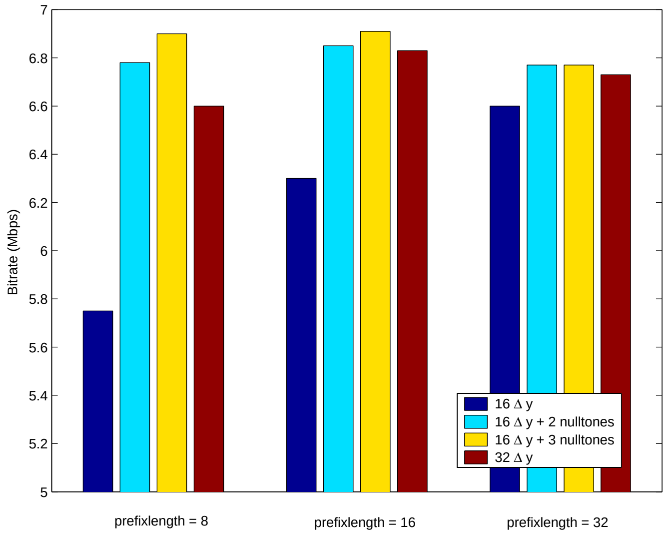

Fig. 2. Performance of several configurations for a 4 km inline channel.

compared: two configurations with only difference terms in y ($T_N = 0$), $T = 16$ and $T = 32$, and two configurations with nulltones, $T = 16$ and $T_N = 2$ or 3. The nulltones are the pilot tone 64 and the unused tones 36 and 37 (36 is not used in the configuration with 2 nulltones). The complexity is (almost) linear in $T+T_N$, so the configuration with $T = 32$ and $T_N = 0$ is approximately twice as complex as the other configurations. We compare the standardized ADSL prefix length $\nu = 32$ with prefix lengths of $\nu = 16$ and $\nu = 8$. Fig. 2 shows the bitrate for all configurations.

For all prefix lengths, the configurations with nulltones outperform the configuration with 32 difference terms. This effect is stronger for shorter prefix lengths. This suggests that it is better to increase $B(z)$ with a few taps than to extend $L_a + 1$ from 16 to 32. Moreover, shorter prefix lengths ($\nu = 8,\ 16$) result in better performance than $\nu = 32$ when using nulltones: reducing the prefix length decreases the overhead with respect to the data rate while, on the other hand, it increases the interblock interference. In fact, cyclic prefix and nulltones are two ways of introducing transmit redundancy that can be traded off to optimize performance.

V. Conclusions

We derived an equalization scheme for DMT-based transmission over IIR channels. We concluded that the per tone equalizer is a close approximation of the optimal MMSE equalizer if certain numerator and denominator conditions are fulfilled. We generalized the per tone equalizer for arbitrary IIR model order. This generalization is based on a suboptimal MMSE criterion. The implementation is very similar to the original per tone equalizer, extended with nulltones, i.e. unused and/or pilot tones. Simulations show that the use of nulltones results in a better performance at a lower complexity. Cyclic prefix and nulltones are two ways to introduce transmit redundancy that can be traded off.

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