JACOBS UNIVERSITY

Mathematics Colloquium at Jacobs University Bremen

Wolf-Jürgen Beyn

(Universität Bielefeld)

will speak on

The Method of Freezing Spatio-Temporal Patterns in Partial Differential Equations

Date: **Monday, April 14, 2008**
Time: **17:15**
Place: **Lecture Hall Research II, Jacobs University**

Abstract:

We consider nonlinear time dependent PDE's on unbounded domains, the solutions of which show specific spatio-temporal patterns. Examples are provided by nonlinear reaction diffusion systems on $\mathbb{R}^d$, such as

$$u_t = \Delta u + f(u),\ u(x,0) = u_0(x), \quad x \in \mathbb{R}^d, t \geq 0, u(x,t) \in \mathbb{R}^m. \tag{1}$$

If the nonlinearity f is of so called excitable type (e.g. a bimodal cubic in case $m = 1$) such systems exhibit travelling or rotating waves for $d = 1$, rigidly rotating or meandering spiral waves for $d = 2$, and scroll waves for $d = 3$.

The *method of freezing* transforms the Cauchy problem (1) into a partial differential algebraic equation (PDAE). The PDAE involves additional algebraic variables that determine a moving coordinate frame in which the patterns become stationary. The extra constraints require the spatial profile to vary in time as little as possible. The method generalizes to evolution equations that are equivariant with respect to the action of a generally noncompact Lie group.

The numerical approximation of the PDAE involves several approximation processes, such as restriction to a bounded domain with asymptotic boundary conditions and discretization in space and time. We show several applications of our method to systems of Ginzburg-Landau and FitzHugh-Nagumo type and we demonstrate how the moving frame freezes specific patterns and prevents them from leaving the computational domain. We discuss some recent work on analytical aspects related to the freezing approach, such as the preservation of asymptotic stability for specific patterns and the influence of numerical approximations on the dicrete and the continuous spectrum of linearizations.

This is joint work with Vera Thümmler (Bielefeld).

Colloquium Tea at ca. 16:45 in the Tea Room of Research II, close to the lecture hall. Everybody is welcome!

M. Stoll